CPSC 314
Final Exam

December 5, 2014

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

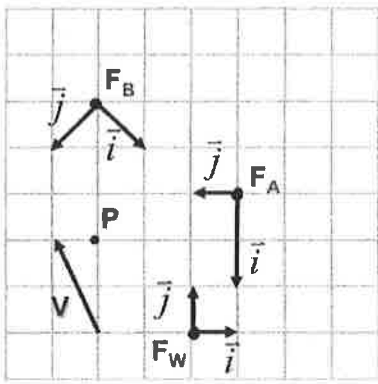
Name: $\boxed{\text{Solutions}}$ ✓

Student Number: ______________________

Question 1	/ 14
Question 2	/ 10
Question 3	/ 5
Question 4	/ 15
Question 5	/ 9
Question 6	/ 8
Question 7	/ 6
TOTAL	/ 67

This exam has 7 questions, for a total of 67 points.

1. Coordinate Frames



$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_W = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_A$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_A = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_B$$

(a) (3 points) Express point P in each of the three coordinate frames.

$P_A(0.5, 3)$

$P_B(1.5, 1.5)$

$P_W(-2, 2)$

(b) (3 points) Express point V in each of the three coordinate frames.

$V_A(-1, 1)$

$V_B(-1.5, -0.5)$

$V_W(-1, 2)$

(c) (2 points) Find the 3×3 homogeneous transformation matrix which takes a point from F_A and expresses it in terms of F_W. I.e., determine M, where $P_W = MP_A$.

$i_A = (0, -2)_W$

$j_A = (-1, 0)_W$

$O_A = (1, 3)_W$

$$\begin{bmatrix} 0 & -1 & 1 \\ -2 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

Check: $P_A(1.5, 1) \Rightarrow P_W(0, 0)$ ✓

(d) (2 points) Find the 3×3 homogeneous transformation matrix which takes a point from F_B and expresses it in terms of F_A. I.e., determine M, where $P_A = MP_B$.

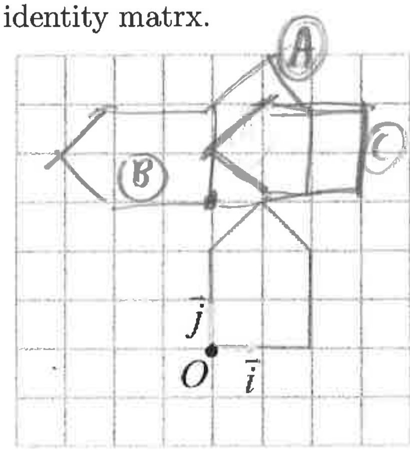
$i_B = (0.5, -1)_A$

$j_B = (0.5, 1)_A$

$O_B = (-1, 3)_A$

$$\begin{bmatrix} 0.5 & 0.5 & -1 \\ -1 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

(e) (3 points) On the grid shown below, sketch the house that would result after each of the three steps in the given sequence of transformations. Label these A, B, C. Assume that the house as shown is drawn with the matrix M initialized to the identity matrx.

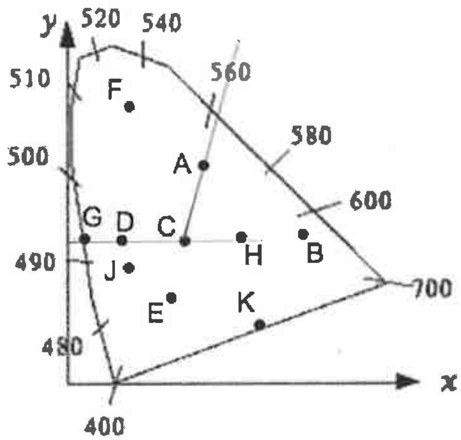


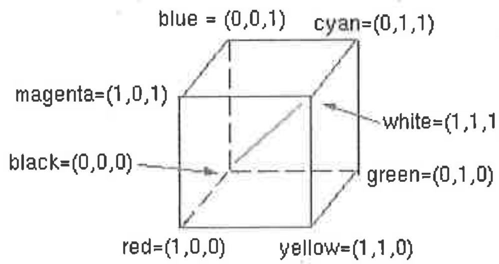

```
A:  M.translate(0,3,0);
B:  M.rotate(90,0,0,1);
C:  M.translate(0,-3,0);
```

(f) (1 point) If the individual transformation matrices given above are labeled M_A, M_B, and M_C, give an expression, in terms of these matrices, for the final compound transformation matrix, M, that is produced by the above sequence.

$M = M_A M_B M_C$

2. Colour





The CIE chromaticity diagram illustrated above has a designated white point, C.

(a) (1 point) What is the dominant wavelength of A?

~562 nm

(b) (1 point) Which of the labeled points have a directly complementary colour to H?

G and D.

(c) (1 point) Which of the labeled points would represent the best choice to use as the three primaries for a colour display?

likely FEB to get the largest gamut.

(d) (1 point) Which point best represents a non-spectral colour?

K

(e) (1 point) Which colours can be mixed with E to produce white?

A

(f) (1 point) What is a **metamer**?

Metamers are two spectra that are different, but that are perceived to be the same, i.e., same colour.

(g) (2 points) What is gamut mapping and when is it needed?

This is the process of mapping desired colours to colours that are within the displayable gamut, for example when printing a photograph.

Where the printer gamut is typically smaller than the camera gamut.

(h) (1 point) What colour would be seen when a cyan surface is illuminated by a yellow light?

cyan = green + blue, ie, surface absorbs red

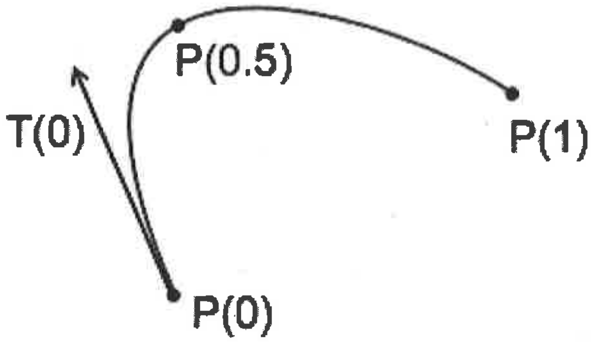
yellow = red + green light. ⇒ $\boxed{\text{green}}$

(i) (1 point) Which of B or H is the more saturated colour?

B: saturation increases as we move away from the white point.

3. (5 points) Parametric Curves

A cubic parametric curve is defined over the interval $t \in [0, 1]$ and is specified by three points, located at $t = \{0, 0.5, 1\}$ and a tangent vector for the start of the curve, i.e., $t = 0$, as shown below.



Develop an expression for $x(t)$ for this kind of curve and give the resulting basis matrix. In your solution, use the following order for the elements in your geometry vector: $x(0), x(0.5), x(1), x'(0)$.

$$x(t) = [t^3 \quad t^2 \quad t \quad 1] \cdot A$$

$$x'(t) = [3t^2 \quad 2t \quad 1 \quad 0] \cdot A$$

$$x(0) = [0 \quad 0 \quad 0 \quad 1] \cdot A$$

$$x(0.5) = [0.5^3 \quad 0.5^2 \quad 0.5 \quad 1] \cdot A$$

$$x(1) = [1 \quad 1 \quad 1 \quad 1] \cdot A$$

$$x'(0) = [0 \quad 0 \quad 1 \quad 0] \cdot A$$

6

$$\begin{bmatrix} x(0) \\ x(0.5) \\ x(1) \\ x'(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0.5^3 & 0.5^2 & 0.5 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$

$$G_x = C \cdot A \Rightarrow A = C^{-1} G_x$$

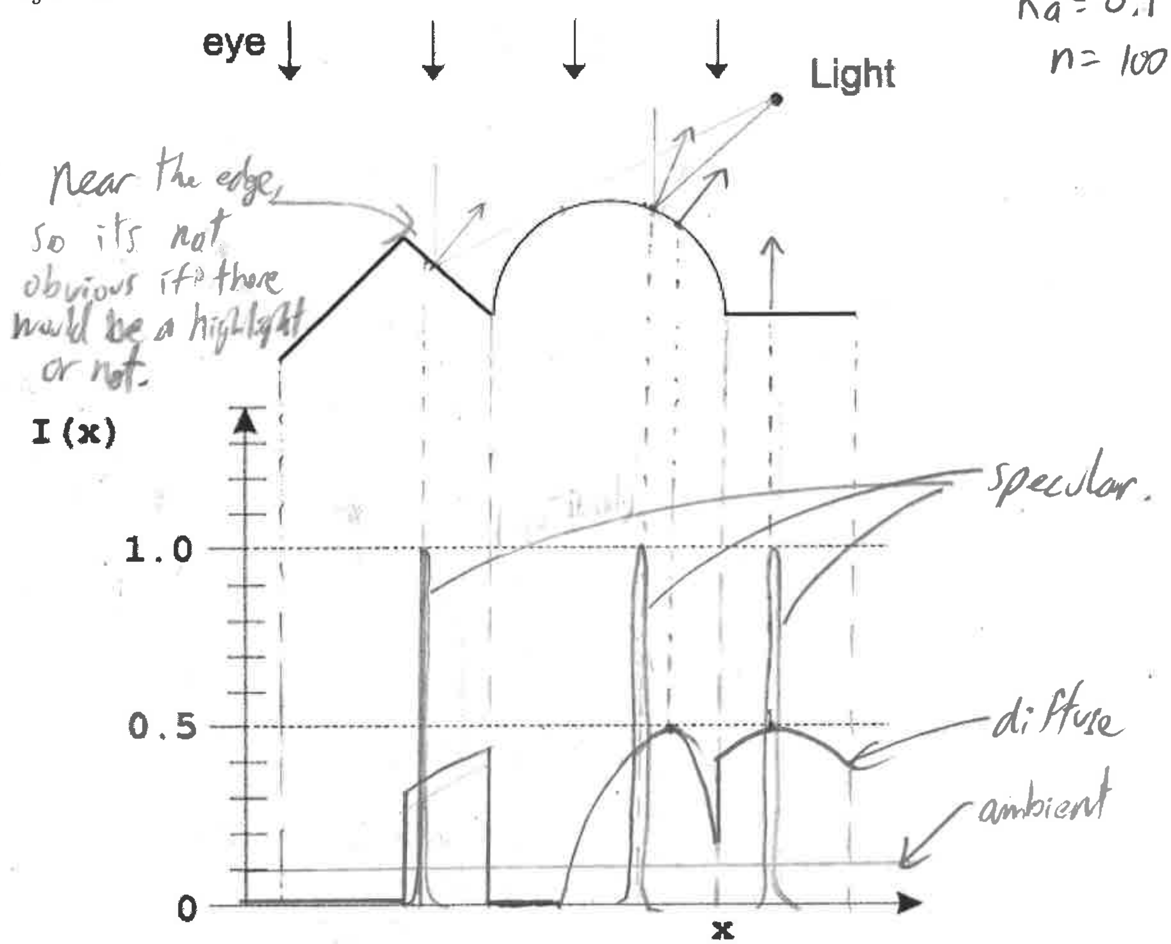
$$x(t) = T \cdot A$$

$$= T \underbrace{C^{-1}}_{\text{basis matrix}} G$$

4. Local Lighting

(a) (6 points) Sketch the ambient, diffuse, and specular components of the illumination for the scene below, as would be computed by the Phong illumination model, i.e., $I = I_a k_a + I_L k_d (N \cdot L) + I_L k_s (R \cdot V)^n$, and using the values $I_a = I_L = 1$, $k_d = 0.5$, $k_s = 1$.

$k_a = 0.1$
$n = 100$



(b) (4 points) Indicate whether each of the variables L, N, R, V are computed or retrieved when implementing Phong shading in a fragment shader. If computed, indicate what is a function of, e.g., Q = f(L,N,R). If retrieved, indicated whether this is from an attribute, varying, or uniform variable. Assume a point light source located at P_L.

L: Computed: $L = normalize(P_L - P)$

N: retrieved: varying variable

R: computed: $R = f(N, L)$

V: computed: $V = normalize(P_{eye} - P_{frag})$
$\hookrightarrow (0,0,0)$ in VCS

(c) (3 points) Describe the details of the Phong model that are not fully captured by the above shading equation, but that need to be considered during implementation.

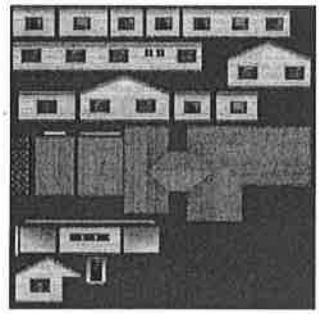
- need to clamp $N \cdot L \geq 0$
- need to clamp $R \cdot V \geq 0$
- could clamp the final sum, and possibly the individual components to be ≤ 1.

(d) (2 points) After making some code changes to a fragment shader in a WebGL application, running the code sometimes produces no visible output on the screen. Describe the next step(s) in determine what went wrong, and give one piece of advice that you would give to others who are beginning to develop shader code for WebGL applications.

- view the javascript console for possible shader compilation errors
- develop shaders incrementally
- verify that vectors are normalized.

5. Texture Mapping

(a) (2 points) Many small pieces of texture are often packed into a single texture map called a *texture atlas*, as shown below. What issues would arise when a texture atlas is used with a standard MIPMAP texture pyramid ?



The filtered levels of the MIPMAP would cause colours to "bleed" between the texture areas and with the given background colour.

(b) (2 points) Why are texture map coordinates commonly defined using normalized coordinates, i.e., $s, t \in [0, 1]$, rather than directly in texel coordinates?

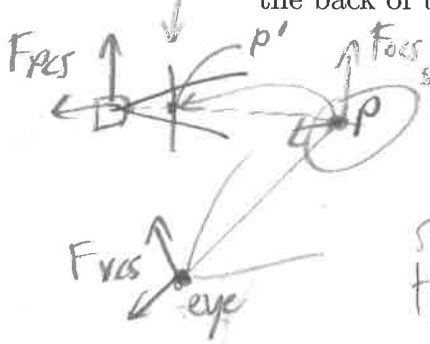
The texture coordinates are independent of the texture map size, which means that it is trivial to use larger or smaller texture maps.

(c) (5 points) Projective texture mapping provides one possible way of automatically assigning texture coordinates to vertices, computed as a function of their position. The process can be thought of as using a virtual slide projector to project the image of the texture map onto objects that are placed in front of the projector and that we want to be textured in this way. In practice, this is computed as follows. Given a point, P, on a target object, a perspective projection is used to project the point onto the image plane for the virtual slide projector, which contains the texture map. The (x, y) location on this image plane can then be used as the (s, t) coordinates that are associated with point P.

Where would you suggest implementing these computations and why? I.e., would this be done on the CPU, the vertex shader, or the fragment shader?

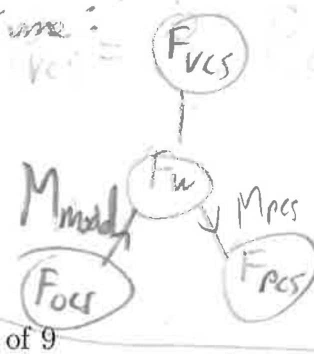
Vertex shader, so that texture coordinates could then simply be interpolated.

Given a point, P_{ocs}, in object coordinates, describe how you would computing the (s, t) coordinates for the point, assuming that you are given a matrix M_{PCS} which takes a point from WCS to the projector coordinate system, PCS. If you need any other information, define it and assume that is available. Continue your answer on the back of this page, as needed.



then:
$s = (x+1)/2$
$t = (y+1)/2$

Assume:



$P_{PCS} = M_{PCS} M_{model} P_{OCS}$

then project, i.e.,

$P'_{PCS} = M_{PCS\,project} P_{PCS}$

then $/h$.

6. Implicit, Explicit, and Parametric Equations

(a) (2 points) Give an implicit equation for a sphere of radius R centred at (a, b, c).

$$F(x,y,z) = R^2 - (x-a)^2 - (y-b)^2 - (z-c)^2$$

(b) (2 points) Give an implicit equation for a 3D plane that has a normal N and that embeds point P_0.

$$F(x,y,z) = Ax + By + Cz + D$$
$$= N \cdot P + D, \text{ where } D = -N \cdot P_0$$

(c) (2 points) Give a parametric equation for a 3D line that passes through points P_1 and P_2.

$$P(t) = P_1 + t P_2$$
or $P(t) = (1-t) P_1 + t P_2$ or ... one solution of many

(d) (2 points) Give a parametric equation for the 3D plane that embeds points P_1, P_2, and P_3.

$$P(s,t) = P_1 + s(P_2 - P_1) + t(P_3 - P_1)$$
one solution of many.

7. Rendering capabilities

(a) (2 points) Give two effects that can be achieved using raytracing that are not easily achieved using projective rendering.

- refraction effects
- specular reflections that could not be modeled with environment maps
- shadows (without needing shadow maps or other tricks)

(b) (2 points) Give a rendering effect that neither projective rendering nor raytracing can easily achieve.

- true global illumination, i.e., multiple diffuse bounces
- caustics, i.e., waves in a pool acting as a ~~lens~~ lens and producing complex patterns on the pool bottom, etc.

(c) (2 points) What is a bidirectional reflection distribution function (BRDF) ?

This is a 4D model of how light from any incoming direction is scattered over the possible outgoing directions. This can be acquired experimentally and serves to model complex appearances, such as satin, velvet, etc.