# CPSC 314 <br> Final Exam 

December 9, 2010

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name: $\qquad$

Student Number: $\qquad$

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| Question 2 | $/ 5$ |
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| Question 7 | $/ 7$ |
| TOTAL | $/ 55$ |

This exam has 7 questions, for a total of 55 points.

1. Coordinate Frames

(a) (2 points) Express point $P$ and vector $V$ in each of the two coordinate frames.
(b) (2 points) Find the $3 \times 3$ homogeneous transformation matrix which takes a point from $F_{3}$ and expresses it in terms of $F_{1}$. I.e., determine $M$, where $P_{1}=M P_{3}$.
2. Transformations
(a) (2 points) Draw the output of the following OpenGL code
// assume transformations initialized to identity glRotatef ( $90,0,0,1$ );
glTranslatef ( $-1,-2,0$ ) ;
glScalef(2,1,1);
glBegin(GL_LINE_LOOP);
glVertex2f $(0,0)$;
glVertex2f $(2,0)$;
glVertex2f(2,3); // asymmetric by design
glVertex2f(0,2);
glEnd();

(b) (2 points) Give the sequence of transformations that would undo the transformations given by: Translate $(a, b, c) \operatorname{Scale}(d, e, f) \operatorname{Rotate}\left(\operatorname{theta}, k_{x}, k_{y}, k_{z}\right)$.
(c) (1 point) What is a more common mechanism for undoing transformations in OpenGL, i.e., reverting to an coordinate frame used in the past?
3. Colour
(a) (2 points) Name five different commonly-used colour spaces.
(b) (1 point) What colour is seen when an LCD projector illuminates a red surface with yellow light? Explain your reasoning.
(c) (1 point) What is a metamer?
(d) (1 point) What is gamut mapping?
(e) (5 points) On the following CIE color diagram, label the colors of the rainbow. Draw the approximate color gamut of an RGB monitor, labeling each of R, G, and B. For the given color A and the whitepoint W , indicate the point D that represents the dominant wavelength of the color A. Indicate the point $\mathrm{D}^{\prime}$ that represents the complementary wavelength.

4. Rasterization and Raytracing
(a) (3 points) Write the pseudocode to rasterize a circle of radius $r$, centered at $(a, b)$, where $r, a, b$ are all given in terms of screen coordinates.
(b) (3 points) Give the equations needed to solve for a ray-triangle intersection.
5. Short Answer
(a) (1 point) Express the point $(2,3,4,4)$, which is given in homogenous coordinates $(x, y, z, h)$, in cartesian coordinates.
(b) (1 point) Give the general form for an implicit line equation in 2D.
(c) (1 point) Give an expression for the function $f(x, y)$ that returns the distance, $d$, to the line $y=x+1$ for any point $(x, y)$.
(d) (2 points) Aaron Barsky gave a guest lecture describing independent game development. What was his opinion as to how to best evaluate and refine a new game idea?
(e) (2 points) Andrew Woo gave a guest lecture about NGRAIN, its tools, and its target market. What is NGRAIN's software used for?
(f) (2 points) Sketch a diagram that illustrates the creation of a shadow with both an umbra and a penumbra. Label these two regions on your diagram.
(g) (2 points) Describe how a shadow map is used to render shadows.
(h) (1 point) What is a caustic?
(i) (1 point) Why type of light paths must be taken into account in order to produce convincing depictions of skin or milk?
(j) (2 points) Briefly summarize the two techniques for modeling curves that were discussed in class.
6. Texture Mapping
(a) (3 points) Write the OpenGL code or equivalent pseudocode to render the wall as shown below. Use GL_QUADS or GL_POLYGON as your primitives. Assume that the given texture map binding has been correctly setup to the shown texture. Both elements on the wall, i.e., the vancouver text and the mountains should come from the texture map.

(b) (2 points) You are looking at a rendered brick wall texture that has been rendered using point sampling. What artifact will appear as you zoom further and further out? What common technique is supported in OpenGL that eliminates this artifact?
(c) (3 points) An environment map uses texture maps to model what is seen in the distance in any given direction. A cube map is one type of environment map - it models the surrounding environment using a large cube that surrounds the scene.
Given a 3D direction $V\left(v_{x}, v_{y}, v_{z}\right)$, develop expressions for the texture coordinates $s$ and $t$, assuming that the ray in direction $V$ hits the given face of the cube as shown below.
For the given face, $s \in[0,1], t \in[0,1]$. Note that the size of the cube in the given diagram does not matter. E.g., $V(k, k,-k)$ will always pass through the $s=1, t=1$ corner of the face regardless of the scale of the cube.

7. Transforming Normals This question looks at how to transform surface normal vectors, which we have not discussed in class.

(a) (2 points) Give the $3 \times 3$ transformation matrix $M$ that implements Scale $(2,1)$, i.e., that scales $x$ by 2 and $y$ by 1 . Transform points $P_{1}$ and $P_{2}$ according to $M$ and sketch the transformed polygon and the new surface normal.
(b) (2 points) A vector can modeled using $h=0$ instead of $h=1$. Transform the normal $N(1,1)$ using the scaling matrix developed above. Does it produce a meaningful answer?
(c) (3 points) The plane equation for a polygon can be given by $N^{T} P=0$, where $N^{T}=\left[\begin{array}{lll}A & B & C\end{array}\right]$ and $P=\left[\begin{array}{lll}x y & z & 1\end{array}\right]^{T}$. The surface normal is given by $(A, B, C)$ and is thus part of $N$. For a point that is transformed by a matrix $M$, i.e., $P^{\prime}=M P$, we wish to solve for the matrix $Q$ that should be used to transform normals, i.e., $N^{\prime}=Q N$. Using the four equations $P^{\prime}=M P, N^{\prime}=Q N, N^{T} P=0$ and $N^{\prime T} P^{\prime}=0$, develop an expression for $Q$.
