CPSC 314
Assignment 3 (6%)

Due Friday March 10, 2017

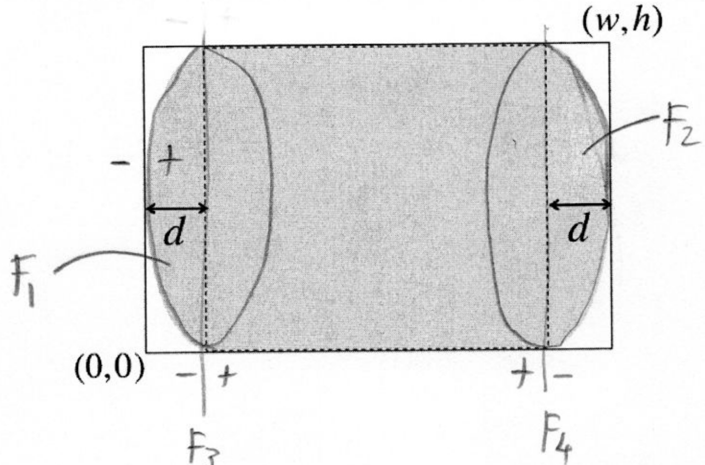
> Answer the questions in the spaces provided on the question sheets. If you run out of space for an answer, use separate pages and staple them to your assignment.

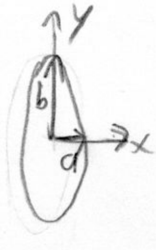
Name: Solutions

Student Number: ____________________

Question 1	/ 5
Question 2	/ 8
Question 3	/ 8
Question 4	/ 4
TOTAL	/ 25

1. (5 points) Scan Conversion
Give the pseudocode for scan converting the solid-shaded region shown below. This consists of a rectangular region of width $w - 2d$ that is augmented by two half ellipses on the left and the right, as shown below. You can assume that $d < w/2$. Use implicit equations to develop your solution.





Ellipse equation:

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

$a = d;\ b = h/2;\ x_{c1} = d;\ y_{c1} = h/2;\ x_{c2} = w - d;\ y_{c2} = h/2;$

for $(x = 0; x \le w; x{+}{+})$ {

  for $(y = 0; y \le h; y{+}{+})$ {

    $F_3 = x - d;$

    $F_4 = w - d - x;$

    if ($F_3 > 0$ and $F_4 > 0$) then

      setPixel(x,y);

    else {

      $F_1 = 1 - \left(\frac{x - x_{c1}}{a}\right)^2 - \left(\frac{y - y_{c1}}{b}\right)^2$

      $F_2 = 1 - \left(\frac{x - x_{c2}}{a}\right)^2 - \left(\frac{y - y_{c2}}{b}\right)^2$

      if ($F_1 > 0$ or $F_2 > 0$) then

        SetPixel(x,y)

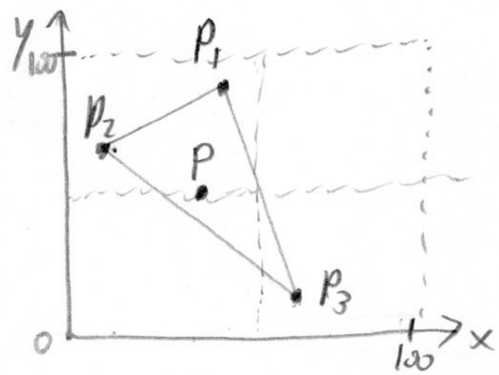
  }

}

2. Scan Conversion and Interpolation

A triangle has device coordinates $P_1(40, 90)$, $P_2(10, 70)$, and $P_3(60, 10)$. You wish to interpolate a value v for point $P(30, 50)$, given the value of v at the vertices: $v_1 = 10, v_2 = 20, v_3 = 60$.

(a) (1 point) Sketch the triangle and the point P.



(b) (3 points) Develop a plane equation for v as a function of x and y. You can use Matlab or an online linear equation calculator (Google this) to solve a set of linear equations for your plane parameters. Compute v for point P using the plane equation.

$$V = Ax + By + C$$

P_1 $\quad 10 = A \cdot 40 + B \cdot 90 + C$

P_2 $\quad 20 = A \cdot 10 + B \cdot 70 + C$

P_3 $\quad 60 = A \cdot 60 + B \cdot 10 + C$

$$\begin{bmatrix} 10 \\ 20 \\ 60 \end{bmatrix} = \begin{bmatrix} 40 & 90 & 1 \\ 10 & 70 & 1 \\ 60 & 10 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

Using online Octave:

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 0.0714 \\ -0.6071 \\ 61.785 \end{bmatrix}$$

$$V = \begin{bmatrix} A & B & C \end{bmatrix} \begin{bmatrix} 30 \\ 50 \\ 1 \end{bmatrix} = \boxed{33.571}$$

(c) (4 points) The barycentric coordinates are defined according to:

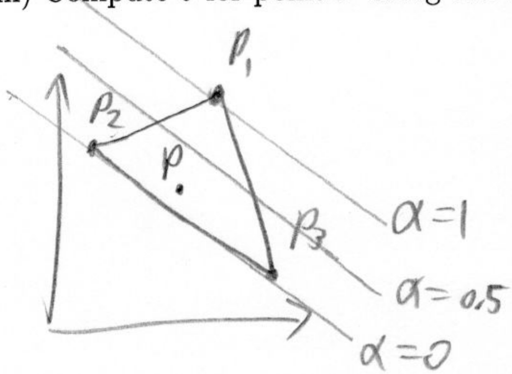
$$P = \alpha P_1 + \beta P_2 + \gamma P_3.$$

(i) Sketch the lines corresponding to $\alpha = 0$, $\alpha = 0.5$, and $\alpha = 1$ on your diagram for part (a).

(ii) Compute the barycentric coordinates for point P.

(iii) Compute v for point P using the Barycentric coordinates.

(i)



(ii) Implicit equation through two points:

$$F(x,y) = Ax + By + C$$

where $A = (y_b - y_a)$, $B = (x_a - x_b)$, $C = y_a x_b - y_b x_a$

$b = 2$ (P_2), $a = 3$ (P_3)

$$F_{23} = (y_2 - y_3)x + (x_3 - x_2)y + y_3 x_2 - y_2 x_3 = 60x + 50y + 100 - 4200$$

$$k = F_{23}(x_1, y_1) = (60)(40) + (50)(90) - 4100 = 2800$$

$$\alpha = \frac{F_{23}(x,y)}{k} = \frac{F_{23}(30,50)}{2800} = \frac{60 \cdot 30 + 50 \cdot 50 - 4100}{2800} = \frac{200}{2800} = \frac{1}{14}$$

$b = 1$ (P_1), $a = 3$ (P_3)

$$F_{13} = (y_1 - y_3)x + (x_3 - x_1)y + y_3 x_1 - y_1 x_3 = 80x + 20y + (10)(40) - (90)(60) = 80x + 20y - 5000$$

$$k = F_{13}(x_2, y_2) = (80)(10) + (20)(70) - 5000 = -2800$$

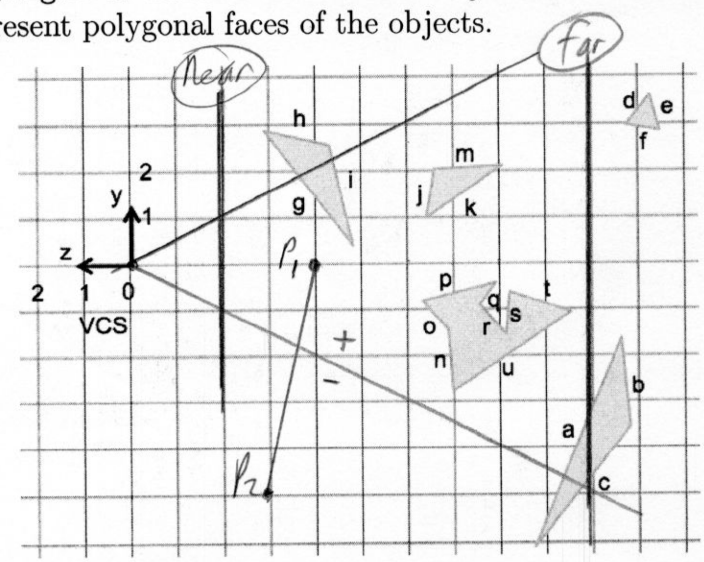
$$\beta = \frac{F_{13}(x,y)}{k} = \frac{(80)(30) + (20)(50) - 5000}{-2800} = \frac{-1600}{-2800} = \frac{4}{7}$$

$$\gamma = 1 - \alpha - \beta = 0.357$$

$$v = \alpha v_1 + \beta v_2 + \gamma v_3 = \left(\frac{1}{14}\right)(10) + \left(\frac{4}{7}\right)(20) + (0.357)60$$

$$\boxed{v = 33.56}$$

3. (8 points) Culling and Clipping
Consider the scene below, shown as a side-view of VCS. `near=2, far=10, bottom=-1, top=1, left=-1,right=1`. Assume that all the objects shown are solid, and that the labelled lines represent polygonal faces of the objects.



(i) On the diagram above, sketch the perspective view volume corresponding to the above parameters. See above figure.

(ii) List the polygons that would be culled by view frustum culling.

b, d, e, f, h

(iii) List the polygons that would be culled by back-face culling. Note: consider both types of culling independently of each other.

b, c, e, h, i, k, m, q, r, t, u

(iv) Sketch the line segment $P_1(0,0,-4)P_2(0,-5,-3)$ on the above diagram. Give the implicit plane equation for the bottom plane of the perspective view volume. Determine the in/out status of P_1 and P_2. Compute the point where the line segment intersects the bottom plane. Give the endpoints of the line segment after it has been clipped to the view volume.

Bottom plane: $y = \frac{z}{2}$ $\qquad F(x,y) = 0 = y - \frac{z}{2}$ which is + for points inside VV

P_1: $F(y_1, z_1) = 0 - \frac{(-4)}{2} = 2$

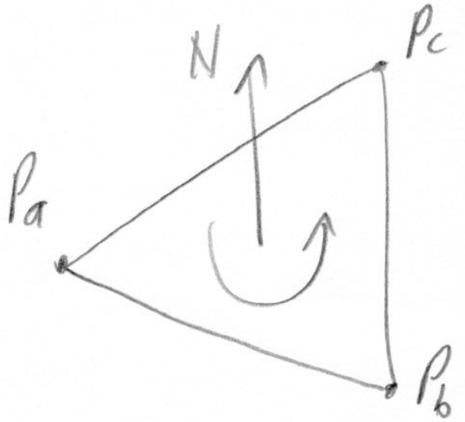
P_2: $F(y_2, z_2) = -5 - \frac{(-3)}{2} = -3.5$

$$t = \frac{-F(P_1)}{F(P_2) - F(P_1)} = \frac{-2}{-3.5-2} = \frac{-2}{-5.5} = 0.36364$$

$P = P_1 + t(P_2 - P_1) \Rightarrow$ $y = 0 + (0.36364)(-5) = -1.8182 = y$, $z = -4 + (0.36364)(1) = -3.6364 = z$ $\Big]\ P$

Clipped line is $\overrightarrow{PP_1}$.

4. (4 points) Suppose that a triangle is defined by three points, $P_a P_b P_c$, as listed in counter-clockwise order when seen from above.
(a) Give the surface normal, N, as expressed using a cross product.
(b) Show how to develop an implicit plane equation, $F(P) = 0$, that embeds the three points.
(c) If a point P' is defined by $P' = P_a + N$, what is the value of $F(P')$? Show how to develop an implicit plane equation that returns the distance to the plane, i.e., $d = F(P)$.



(a) $N = (P_b - P_a) \times (P_c - P_a)$

(b) $F(P) = N \cdot P + D$

$0 = N \cdot P_a + D$ substitute any point.

$\Rightarrow D = -N \cdot P_a$

(c) $F(P) = N \cdot P + D$

$= N \cdot (P_a + N) - N \cdot P_a$

$= N \cdot P_a + N \cdot N - N \cdot P_a$

$= N \cdot N$