Ref

CPSC 314
Assignment 2 Theory

due: Monday, January 30, 2017, 11:59pm
Worth 5% of your final grade.

Answer the questions in the spaces provided on the question sheets.

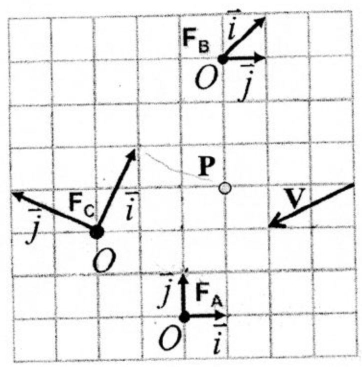
Name: ______________________________

Student Number: ______________________________

Question 1	/ 15
Question 2	/ 8
Question 3	/ 9
Question 4	/ 11
Question 5	/ 4
TOTAL	/ 47

CPSC 314 Assignment 2 January 2017

1. Transformations as a change of coordinate frame



$$\begin{bmatrix}x\\y\\1\end{bmatrix}_A = \begin{array}{c} \begin{array}{ccc} i_B & j_B & O_B \end{array} \\ \begin{bmatrix}1&1&1\\1&0&6\\0&0&1\end{bmatrix} \end{array} \begin{bmatrix}x\\y\\1\end{bmatrix}_B$$

Express basis vectors and origin of F_B wrt F_A

$$\begin{bmatrix}x\\y\\1\end{bmatrix}_C = \begin{array}{c} \begin{array}{ccc} i_A & j_A & O_A \end{array} \\ \begin{bmatrix}0.2&0.4&-0.4\\-0.4&0.2&-1.2\\0&0&1\end{bmatrix} \end{array} \begin{bmatrix}x\\y\\1\end{bmatrix}_A$$

See also Q1(b) for i_A, j_A

Express basis vectors and origin of F_A wrt F_C

$O_A = 2i_A - 2j_A = 2(0.2i_c - 0.4j_c) - 2(0.4i_c + 0.2j_c) = -0.4i_c - 1.2j_c$

(a) (3 points) Express the coordinates of point P with respect to coordinate frames A, B, and C.

$\boxed{P_A(1,3)} = O_A + 1\vec{i}_A + 3\vec{j}_A$

$\boxed{P_B(-3,3)} = O_B - 3\vec{i}_B + 3\vec{j}_B$

$\boxed{P_C(1,-1)} = O_c + 1\vec{i}_c - 1\vec{j}_c$

(b) (3 points) Express the coordinates of vector V with respect to coordinate frames A, B, and C.

$V_A = -2\vec{i}_A - 1\vec{j}_A = \boxed{V_A\langle -2,-1\rangle} \rightarrow V_c = -2(0.2i_c - 0.4j_c) - 1(0.4i_c + 0.2j_c) = -0.8i_c + 0.6j_c$

$\boxed{V_c\langle -0.8, 0.6\rangle}$

$V_B = -1\vec{i}_B - 1\vec{j}_B = \boxed{V_B\langle -1,-1\rangle}$

V_c: easier to first express in terms of grid.

$5i_A = i_c - 2j_c \Rightarrow i_A = 0.2i_c - 0.4j_c$

$5j_A = 2i_c + j_c \quad j_A = 0.4i_c + 0.2j_c$

(c) (3 points) Fill in the 2D transformation matrix that takes points from F_B to F_A, as given to the right of the above figure.

(d) (3 points) Fill in the 2D transformation matrix that takes points from F_A to F_C, as given to the right of the above figure. Hint: You might first want to determine what combination of i_C and j_C are needed to reproduce $5i_A$ and $5j_A$.

(e) (3 points) Using your answers for the above two matrices, develop a 2D transformation matrix that takes points from F_B to F_C. Test your solution using point P.

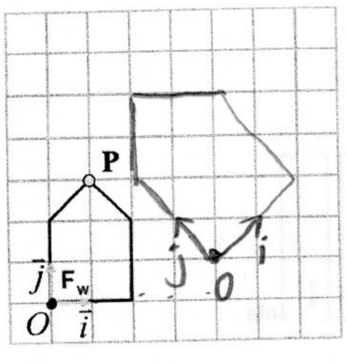
$$P_c = M_{A\to C} M_{B\to A} P_B$$

$$\begin{bmatrix}x\\y\\1\end{bmatrix}_A = \underbrace{\begin{bmatrix}0.2&0.4&-0.4\\-0.4&0.2&-1.2\\0&0&1\end{bmatrix}\begin{bmatrix}1&1&1\\1&0&6\\0&0&1\end{bmatrix}}\begin{bmatrix}x\\y\\1\end{bmatrix}_B$$

matches P_c given ✓

$$\begin{bmatrix}1\\-1\\1\end{bmatrix}_c = \begin{bmatrix}-1.8+0.6+2.2\\0.6-1.2-0.4\\1\end{bmatrix} = \begin{bmatrix}0.6&0.2&2.2\\-0.2&-0.4&-0.4\\0&0&1\end{bmatrix}\begin{bmatrix}-3\\3\\1\end{bmatrix}_B$$

2. Interpreting a Matrix Transformation



$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_w = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{obj}$$

(a) (2 points) On the above diagram, sketch the origin and basis vectors of the coordinate frame F_{obj} that results from the given transformation matrix.

(b) (2 points) The drawing of the above house represents its untransformed shape. Sketch the transformed version of the house in the above diagram.

(c) (2 points) Given $M = Translate(a, b, 0)Rotate(z, \theta)Scale(c, c, c)$, provide the values of a, b, c and θ that would implement the given transformation.

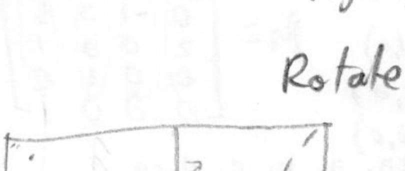
Think about these operations in local coords for L-to-R

$Translate(4,1,0)\ Rotate(z,45^\circ)\ Scale(\sqrt{2},\sqrt{2},\sqrt{2})$

(d) (2 points) Given $M = Rotate(z, \theta)Translate(a, b, 0)Scale(c, c, c)$, provide the values of a, b, c and θ that would implement the given transformation.

Again, think L-to-R, local coords:

$Rotate(z,45^\circ)\ Translate\left(\frac{5}{\sqrt{2}}, -\frac{3}{\sqrt{2}}, 0\right)\ Scale(\sqrt{2},\sqrt{2},\sqrt{2})$.



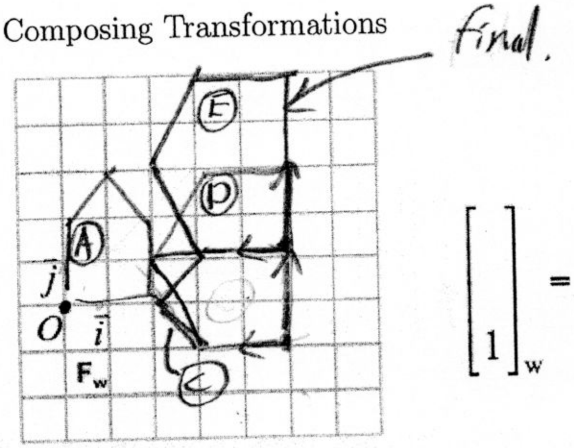
$$4 i_w + j_w = 4\left(\frac{i-j}{\sqrt{2}}\right) + 1\left(\frac{i+j}{\sqrt{2}}\right)$$

$$= \frac{5i-3j}{\sqrt{2}} = \begin{bmatrix} 5/\sqrt{2} \\ -3/\sqrt{2} \end{bmatrix}$$

$i_w = \frac{i-j}{\sqrt{2}}$

$j_w = \frac{i+j}{\sqrt{2}}$

3. Composing Transformations



$$\begin{bmatrix} \\ \\ \\ 1 \end{bmatrix}_w = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} \\ \\ \\ 1 \end{bmatrix}_{obj}$$

(a) (3 points) Consider a house in the xy-plane, defined by the coordinates (0,0), (2,0), (2,2), (1,3), 0,2). Assume $z = 0$ for all vertices.
Sketch the house resulting from the following transformations. Assume that the matrix M is initialised to the identity matrix.

```
A M.identity();
B M.translate(5,-1,0);
C M.rotate(+90, 0,0,1);
D M.scale(2,1,1);
E M.translate(1,0,0);
  drawHouse();
```

(b) (3 points) Give the resulting 4×4 transformation matrix, M.

We could multiply all the matrices: $M = M_A M_B M_C M_D M_E$

$$M = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & 5 \\ & 1 & & -1 \\ & & 1 & 0 \\ & & & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & 1 \\ & 1 & & 0 \\ & & 1 & 0 \\ & & & 1 \end{bmatrix}$$

Or: directly build the matrix M from the origin and basis vectors, from the diagram

$O_{obj} = (5,1,0)$
$i_{obj} = (0,2,0)$
$j_{obj} = (-1,0,0)$

$$M = \begin{array}{c} \begin{array}{cccc} i & j & k & O \end{array} \\ \begin{bmatrix} 0 & -1 & 0 & 5 \\ 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(c) (3 points) What values would need to be assigned to `theta, a, b, c, d, e, f` in order for the following transformations to yield an identical final transformation?

L-to-R, local coords

```
A M.identity();
B M.rotate(theta, 0,0,1);
C M.translate(a,b,c);
D M.scale(d,e,f);
  drawHouse();
```

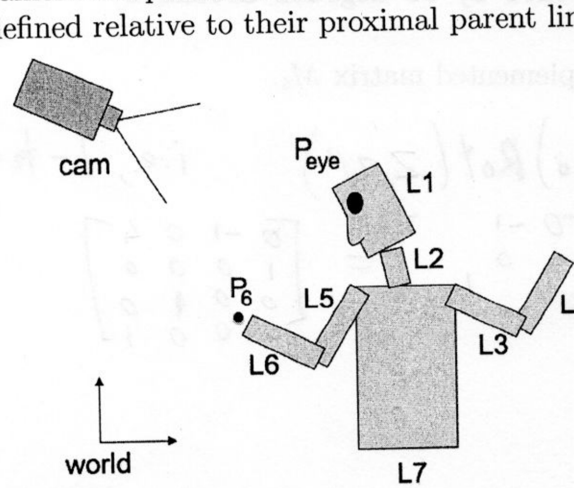
$M = M_A M_B M_C M_D$

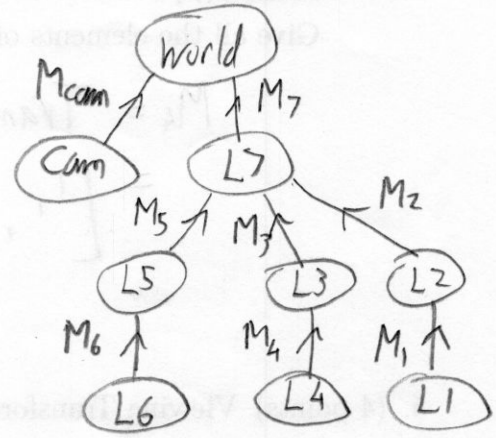
Rotate$(z, 90°)$ Translate$(1,-5,0)$ Scale$(2,1,0)$.

$\theta = 90°$
$(a,b,c) = (1,-5,0)$
$(d,e,f) = (2,1,0)$

4. Scene Graphs

(a) (3 points) Sketch a scene graph for the scene below. Use directed edges, i.e., arrows, to represent transformations that connect the nodes in the graph. Label each edge with a unique name, M_n, which indicates the transformation matrix that takes points from the frame F_n (for link Ln) to its parent frame. Use the world coordinate frame as the root note of the scene graph. Assume that the body (L7) and the camera are positioned relative to the world frame. Assume that all other parts are defined relative to their proximal parent links, i.e., the links closer to the body.





(b) (2 points) Give an expression for the composite transformation that would be used when drawing point P_6, as defined in frame F_6 for link L6. i.e., it should transform point P_6 to the camera coordinate frame. Your answer should be expressed as a product of the matrices used to label your scene graph.

$$P_{cam} = M_{cam}^{-1} M_7 M_5 M_6 P_6$$

(c) (2 points) Give an expression for the composite transformation that transforms point, P_6, as defined in frame F_6, to the head coordinate frame, F_1. Your answer should be expressed as a product of the matrices used to label your scene graph.

$$P_{head} = M_1^{-1} M_2^{-1} M_5 M_6 P_6$$

$$P_6 = M_6^{-1} M_5^{-1} M_2 M_1 P_1$$

(d) (2 points) Give an expression for the opposite transformation, i.e., one that transforms a point, P_1, in the head coordinate frame, F_1, to the coordinate frame F_6.

(e) (2 points) Give the value of the matrix M_4 that is implemented by the code below, i.e., the relative transformation of L4 with respect to L3.

```
   // implement M4: transformation of L4 relative to L3
M.translate(4,0,0);    // translate in x
M.rotate(90,0,0,1);  // rotate by 90 degrees around z
DrawL4();
```

Give all the elements of the implemented matrix M_4.

$$M_4 = Trans(4,0,0)\, Rot(z, 90^\circ) \qquad \text{i.e., L-to-R, local coords.}$$

$$= \begin{bmatrix} 1 & & & 4 \\ & 1 & & 0 \\ & & 1 & 0 \\ & & & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & & \\ 1 & 0 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 4 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. (4 points) Viewing Transformation

Determine the viewing transformation, M_{view}, that takes points from WCS (world coordinates) to VCS (viewing or camera coordinates), for the following camera parameters: $P_{eye} = (-50, 20, 20)$, $P_{ref} = (0, 20, 20)$, $V_{up} = (1, 0, 1)$. There is no need to numerically invert any matrices, i.e., it is ok to specify $M_{view} = M_{cam}^{-1}$.

P_{eye} $\quad \vec{k} \quad$ P_{ref}

$$\vec{k} = \frac{P_{eye} - P_{ref}}{\| P_{eye} - P_{ref} \|} = \frac{\langle -50, 0, 0 \rangle}{\| \langle -50, 0, 0 \rangle \|} = \langle -1, 0, 0 \rangle$$

$$\vec{i} = \frac{\vec{V}_{up} \times \vec{k}}{\| \sim \|} = \langle 1, 0, 1 \rangle \times \langle -1, 0, 0 \rangle$$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ -1 & 0 & 0 \end{vmatrix} = i(0) - j(1) + k(0) = \langle 0, -1, 0 \rangle$$

$$\vec{j} = k \times i = \langle -1, 0, 0 \rangle \times \langle 0, -1, 0 \rangle = \langle 0, 0, 1 \rangle$$

$$M_{cam} = \begin{bmatrix} 0 & 0 & -1 & -50 \\ -1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad M_{view} = M_{cam}^{-1}$$