Perspective Projection

- project all geometry
  - through common center of projection (eye point)
  - onto an image plane

Real Cameras

- pinhole camera has small aperture (lens opening)
- minimize blur
- problem: hard to get enough light to expose the film
- solution: lens
  - permits larger apertures
  - permits changing distance to film plane without actually moving it
  - cost: limited depth of field where image is in focus

Graphics Cameras

- real pinhole camera: image inverted
- computer graphics camera: convenient equivalent

Perspective Projection

- our camera must model perspective

Pinhole Camera

- non-zero sized hole
- blur: rays hit multiple points on film plane

General Projection

- image plane need not be perpendicular to view plane

Projections I

- viewing 2

Pinhole Camera

- theoretical perfect pinhole
- light shining through tiny hole into dark space yields upside-down picture

Projective Transformations

- properties
  - lines mapped to lines and triangles to triangles
  - parallel lines do NOT remain parallel
    - e.g. rails vanishing at infinity
  - affine combinations are NOT preserved
    - e.g. center of a line does not map to center of projected line (perspective foreshortening)

Basic Perspective Projection

- desired result for a point \([x, y, z, 1]^T\) projected onto the view plane:
  \[
  \begin{align*}
  x' &= \frac{x}{d} = \frac{x - c}{z} \\
  y' &= \frac{y}{d} = \frac{y - c}{z} \\
  z' &= d \\
  \end{align*}
  \]
- nonuniform foreshortening
- not affine

Counterexamples?

- similar triangles
Simple Perspective Projection Matrix

\[
\begin{bmatrix}
\frac{x}{z/d} \\
\frac{y}{z/d} \\
\frac{z}{d}
\end{bmatrix}
\]

is homogenized version of

\[
\begin{bmatrix}
x \\
y \\
z \\
d
\end{bmatrix}
\]

where \( w = z/d \)

Perspective Projection

- expressible with 4x4 homogeneous matrix
- use previously untouched bottom row
- perspective projection is irreversible
- many 3D points can be mapped to same \((x, y, d)\) on the projection plane
- no way to retrieve the unique \(z\) values

Orthographic Camera Projection

- camera's back plane parallel to lens
- infinite focal length
- no perspective convergence
- just throw away \(z\) values

\[
\begin{bmatrix}
x' \\
y' \\
z' \\
1
\end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ d \end{bmatrix}
\]

Perspective to Orthographic

- transformation of space
- center of projection moves to infinity
- view volume transformed
- from frustum (truncated pyramid) to parallelepiped (box)

Moving COP to Infinity

- as COP moves away, lines approach parallel
- when COP at infinity, orthographic view

Asymmetric Frusta

- our formulation allows asymmetry
- why bother?

Asymmetric Frusta

- our formulation allows asymmetry
- why bother? binocular stereo
- view vector not perpendicular to view plane

Simpler Formulation

- left, right, bottom, top, near, far
  - nonintuitive
  - often overkill
  - look through window center
  - symmetric frustum
  - constraints
  - left = -right, bottom = -top

Field-of-View Formulation

- FOV in one direction + aspect ratio (w/h)
- determines FOV in other direction
- also set near, far (reasonably intuitive)

Demos

- frustum
  - http://webglfundamentals.org/webgl/frustum-diagram.html

- orthographic vs projection cameras
  - http://threejs.org/examples/#webgl_canvas_camera_orthographic2
  - http://threejs.org/examples/webgl_camera