L3. Points and Vectors

Objectives: - Distinguish between points and vectors
  - How to represent these
    • In a computer program (coordinates)
    • In mathematics (notation)

A point is a location in space. If you know a distinguished point, which we can call "origin"

\[ \vec{O} + \vec{p} \]

\[ \vec{O} \text{ is a displacement vector} \]

\[ \vec{p} = \vec{O} + \vec{v} \]

There are other kinds of vectors: velocity, normals, ...
Vector space

\[ V = \sum_{i=1}^{\infty} a_i b_i, \quad \text{if } a_i, b_i \in V \]
\[ \lambda a_i \in V, \quad \text{if } a_i \in V \]

Basis \( \{ b_1, b_2, \ldots \} \)

for any \( \vec{v} = \alpha_1 b_1 + \alpha_2 b_2 \)

coordinates of \( \vec{v} \) in the basis

A basis is a linearly independent set of vectors, which is complete for \( V \).
The size of the basis is called the dimension of \( V \).
The basis is not unique.

Why do we need this? Given no coordinates

Orthonormal basis

Suppose we have a "dot" product \( \vec{v}_1 \cdot \vec{v}_2 \) scalar

can define \( \vec{v}_1 \cdot \vec{v}_2 = ||v_1|| ||v_2|| \cos \theta \), where \( ||v_i|| \) is a norm

can "normalize" a vector, i.e.,
\[ \frac{\vec{v}}{||v||} = \frac{1}{||v||} \vec{v} \]
Two vectors are orthogonal if \( \vec{a} \cdot \vec{b} = 0 \).

A basis in which all vectors are mutually orthogonal, and have norm = 1, is called "orthonormal".

Dot product of 2 vectors in an orthonormal basis has a simple form:

\[
\begin{align*}
\vec{u} &= u_1 \vec{b}_1 + u_2 \vec{b}_2 \\
\vec{v} &= v_1 \vec{b}_1 + v_2 \vec{b}_2 \\
\vec{u} \cdot \vec{v} &= u_1 v_1 \vec{b}_1 \cdot \vec{b}_1 + u_1 v_2 \vec{b}_1 \cdot \vec{b}_2 + \cdots \\
&= u_1 v_1 + u_2 v_2
\end{align*}
\]

This is why orthonormal is great, we'll use it whenever possible.

Important Notation

<table>
<thead>
<tr>
<th>Important</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>Point</td>
</tr>
<tr>
<td>( \vec{v} )</td>
<td>Vector</td>
</tr>
<tr>
<td>( \vec{a} )</td>
<td>Column Matrix</td>
</tr>
<tr>
<td>( a )</td>
<td>Row Matrix</td>
</tr>
</tbody>
</table>

Just involves scalars!

S. Change from Book.

\[
\begin{bmatrix}
a_1 \\
a_2 \\
a_3
\end{bmatrix}
\]