Perspective Projection

Perspective Cameras and Projective Transformation

- What do you get when you call
  var camera = new
  THREE.PerspectiveCamera(30, 1, 0.1, 1000);
  // view angle, aspect ratio, near, far
2D Perspective (x behave like y in 3D)

\[
\begin{pmatrix}
\frac{y}{2} \\
1
\end{pmatrix}
\Rightarrow
\begin{pmatrix}
\frac{-y}{2} \\
1
\end{pmatrix}
\]

Want to represent the light ray through the origin "central projection"

\[
\text{Pin hole camera interpretation}
\]

A scene as window

To find \( y_p \), use similar triangles

\[
\frac{y_p}{y} = -\frac{1}{2} \quad \text{or} \quad y_p = -\frac{y}{2}
\]

This is a non-linear transformation of space!

§ Representing this using a homogeneous transformation (i.e. \( n+1 \times n+1 \) matrix, where \( n \) is dimension)

\[
\begin{pmatrix}
y \\
z \\
1
\end{pmatrix}
\Rightarrow
\begin{pmatrix}
\frac{-y}{2} \\
z \\
1
\end{pmatrix}
= -\frac{1}{2}
\begin{pmatrix}
y \\
z \\
1
\end{pmatrix}
\equiv
\begin{pmatrix}
y \\
z \\
1
\end{pmatrix}
\]

\[
\text{as}\quad \text{gl-perspective}
\]

From now on, identify all non-zero multiples of a point with itself. That is, we ignore the "scale"
Dividing by the last entry \( \frac{w}{w} \) is called homogenization.

Using this we can write projection as a matrix! \[
\mathbf{p}_b = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
-1 & 0
\end{bmatrix}
\] achieves the projection above.

But all depth info is lost. The matrix is singular.

A better projection matrix that retains some depth info.
\[
\mathbf{p}_u = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
-1 & 0 & 0
\end{bmatrix}
\] sometimes called an "unhinging" transform.

What does this do to
\[
\begin{bmatrix}
y \\
z \\
1
\end{bmatrix}
\]

\[
\begin{bmatrix}
y \\
1
\end{bmatrix}
\]

\[
\begin{bmatrix}
y \\
-1/2 \\
1
\end{bmatrix}
\]

Converges lines through the origin to parallel lines.

\[
\mathbf{p}_u
\]

\[
\mathbf{p}_o
\]

So this is now an orthographic projection.

So the total perspective matrix is \( \mathbf{P} = \mathbf{p}_o \mathbf{p}_u \).
PerspectiveCamera
Eye coords → Clip coords

\[
\begin{bmatrix}
\frac{1}{\alpha \tan\left(\frac{\theta}{2}\right)} & 0 & 0 & 0 \\
0 & \frac{1}{\tan\left(\frac{\theta}{2}\right)} & 0 & 0 \\
0 & 0 & \frac{f+n}{f-n} & -\frac{2fn}{f-n} \\
0 & 0 & -1 & 0
\end{bmatrix}
\]