Projection and Rasterization Redux

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Partly from
Textbook Chapters 10 and 12

Midterm 2 update

- Textbook. Read **ALL** of these, except as noted
  - Ch 14 Materials (shading and lighting)
  - Ch 15 Texture Mapping
  - Ch 3.6 (transformation of normals)
  - Ch 9 Interpolation. Skip 9.2 and 9.3
  - Ch 10 Projection
  - Ch 12 From Vertex to Pixel
  - **Ch 11:** We’ll cover this AFTER midterm, so Wed. will be review
How many control points are there for a segment of a Bezier curve of degree 3?

a) 1  

b) 2  

c) 3  

d) 4  

e) None of the above

If you use 4 points $C_0 = (0,0,0)$, $C_1 = (1,0,0)$, $C_2 = (0,1,0)$, $C_3 = (0,0,1)$ as the control points for a piece of Bezier curve, what is its tangent direction at $C_0$?

a) $(1,0,0)$  

b) $(0,1,0)$  

c) $(0,0,1)$  

d) None of the above
Recap basic pinhole projection

Basic projection matrix

\[ P_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

Check \( \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \)

\[ P_b \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} \]

Homogeneous

\[ \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} \]

Our initial conception of 4x4 matrices & homogeneous conds. was a way to combine points \( \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \) and vectors \( \begin{pmatrix} x \\ y \\ z \\ 0 \end{pmatrix} \)

Now we are using all 4 conds., e.g. \( w \)

\[ \text{Note: book often units} \quad \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \]
What we have really done is to model $\mathbb{R}^3$ as a **projective space** $(\mathbb{P}^3)$.

A projective transform is any non-singular $4 \times 4$ matrix.

Let's make a better projection

$$P_p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Non-singular!

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \xrightarrow{P_p} \begin{pmatrix} x \\ y \\ z \\ -1 \end{pmatrix} \xrightarrow{H} \begin{pmatrix} -x/2 \\ -y/2 \\ 1/2 \\ 1 \end{pmatrix}$$

So e.g.

$$\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \xrightarrow{P_p} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \xrightarrow{H} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

because a point at infinity.

Origin

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \xrightarrow{P_p} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \xrightarrow{H} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

became a point!

Vector $\begin{pmatrix} 0 \\ y \\ -1/6 \end{pmatrix} \xrightarrow{P_p} \begin{pmatrix} 0 \\ y \\ -1 \\ 0 \end{pmatrix} \xrightarrow{H} \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}$

became a point!
In a projective space, \((\frac{x}{y} \ 0)\) is a point at infinity in the direction of that \(z\) vector.

Simplifies a lot of things, e.g. any two lines intersect at a point, parallel lines

\[ \text{horizon} \]

What this does to 3D space:

What gluh: Perspective (normalised device coord)
Pipeline \[\xrightarrow{\text{model warp}}\] Model \[\xrightarrow{\text{view}}\] Eye Coord \[\xrightarrow{\text{projection}}\] Chip Coord \[\left(\begin{array}{c} x_n \\ y_n \\ w_n \\ w_n \end{array}\right)\] \[\begin{array}{c} -w_c \leq x_c \leq w_c \\ -1 \leq x_n \leq 1 \end{array}\] Instead of chip \(-w_c \leq x_c \leq w_c\)