University of British Columbia
CPSC 314 Computer Graphics
Jan-Apr 2013

Tamara Munzner

Viewing

http://www.ugrad.cs.ubc.ca/~cs314/Vjan2013

Reading for This Module

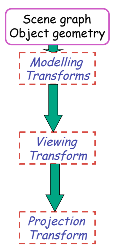
- FCG Chapter 7 Viewing
- FCG Section 6.3.1 Windowing Transforms
- RB rest of Chap Viewing
- RB rest of App Homogeneous Coords
- RB Chap Selection and Feedback
- RB Sec Object Selection Using the Back Buffer
  - (in Chap Now That You Now)

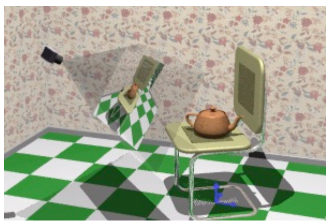
Viewing

Using Transformations

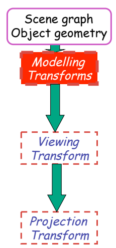
- three ways
  - modelling transforms
    - place objects within scene (shared world)
    - affine transformations
  - viewing transforms
    - place camera
    - rigid body transformations: rotate, translate
  - projection transforms
    - change type of camera
    - projective transformation

Rendering Pipeline

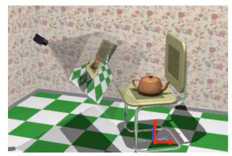




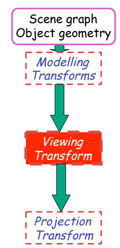
Rendering Pipeline



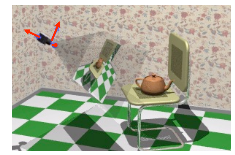
- result
  - all vertices of scene in shared 3D world coordinate system



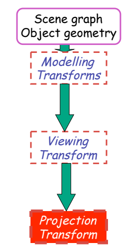
Rendering Pipeline



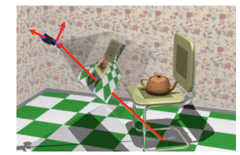
- result
  - scene vertices in 3D view (camera) coordinate system



Rendering Pipeline



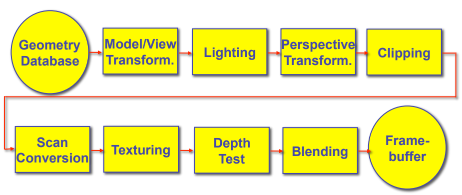
- result
  - 2D screen coordinates of clipped vertices



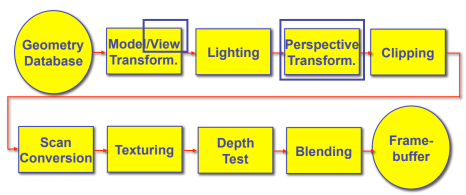
Viewing and Projection

- need to get from 3D world to 2D image
- projection: geometric abstraction
  - what eyes or cameras do
- two pieces
  - viewing transform:
    - where is the camera, what is it pointing at?
  - perspective transform: 3D to 2D
    - flatten to image

Rendering Pipeline



Rendering Pipeline



OpenGL Transformation Storage

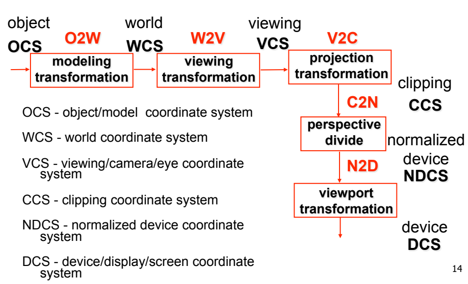
- modeling and viewing stored together
  - possible because no intervening operations
- perspective stored in separate matrix
- specify which matrix is target of operations
  - common practice: return to default modelview mode after doing projection operations

```
glMatrixMode(GL_MODELVIEW);
glMatrixMode(GL_PROJECTION);
```

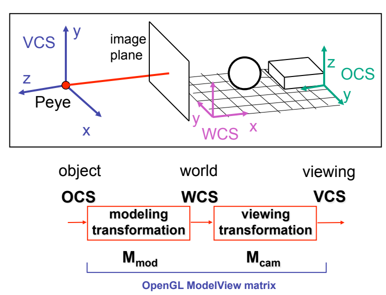

Coordinate Systems

- result of a transformation
- names
  - convenience
    - animal: leg, head, tail
  - standard conventions in graphics pipeline
    - object/modelling
    - world
    - camera/viewing/eye
    - screen/window
    - raster/device

Projective Rendering Pipeline



Viewing Transformation



Basic Viewing

- starting spot - OpenGL
  - camera at world origin
    - probably inside an object
  - y axis is up
  - looking down negative z axis
    - why? RHS with x horizontal, y vertical, z out of screen
- translate backward so scene is visible
  - move distance d = focal length
- where is camera in P1 template code?
  - 5 units back, looking down -z axis

Convenient Camera Motion

- rotate/translate/scale versus
  - eye point, gaze/lookat direction, up vector
  - demo: Robins transformation, projection

OpenGL Viewing Transformation

```
gluLookAt(ex,ey,ez,lx,ly,lz,ux,uy,uz)
```

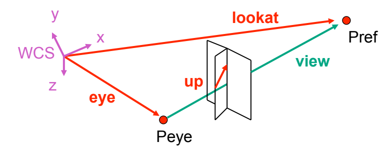
- postmultiplies current matrix, so to be safe:

```
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
gluLookAt(ex,ey,ez,lx,ly,lz,ux,uy,uz)
// now ok to do model transformations
```

- demo: Nate Robins tutorial *projection*

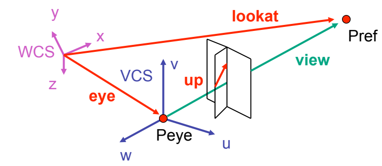
Convenient Camera Motion

- rotate/translate/scale versus
  - eye point, gaze/lookat direction, up vector



Placing Camera in World Coords: V2W

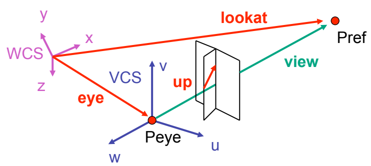
- treat camera as if it's just an object
  - translate from origin to **eye**
  - rotate **view** vector (**lookat** – **eye**) to **w** axis
  - rotate around **w** to bring **up** into **vw**-plane



Deriving V2W Transformation

- translate origin to **eye**

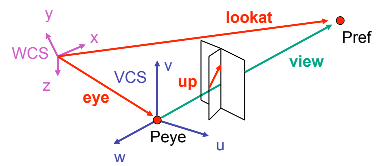
$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Deriving V2W Transformation

- rotate **view** vector (**lookat** – **eye**) to **w** axis
  - **w**: normalized opposite of **view/gaze** vector **g**

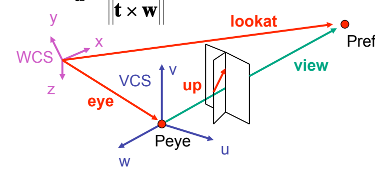
$$\mathbf{w} = -\hat{\mathbf{g}} = -\frac{\mathbf{g}}{\|\mathbf{g}\|}$$



Deriving V2W Transformation

- rotate around **w** to bring **up** into **vw**-plane
  - **u** should be perpendicular to **vw**-plane, thus perpendicular to **w** and **up** vector **t**
  - **v** should be perpendicular to **u** and **w**

$$\mathbf{u} = \frac{\mathbf{t} \times \mathbf{w}}{\|\mathbf{t} \times \mathbf{w}\|} \qquad \mathbf{v} = \mathbf{w} \times \mathbf{u}$$



Deriving V2W Transformation

- rotate from WCS **xyz** into **uvw** coordinate system with matrix that has columns **u**, **v**, **w**

$$\mathbf{u} = \frac{\mathbf{t} \times \mathbf{w}}{\|\mathbf{t} \times \mathbf{w}\|} \qquad \mathbf{v} = \mathbf{w} \times \mathbf{u} \qquad \mathbf{w} = -\hat{\mathbf{g}} = -\frac{\mathbf{g}}{\|\mathbf{g}\|}$$

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad M_{V2W} = TR$$

- reminder: rotate from **uvw** to **xyz** coord sys with matrix **M** that has columns **u,v,w**

V2W vs. W2V

- $M_{V2W}=TR$

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- we derived position of camera as object in world
  - invert for gluLookAt: go from world to camera!
- $M_{W2V}=(M_{V2W})^{-1}=R^{-1}T^{-1}$

$$\mathbf{R}^{-1} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{T}^{-1} = \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - inverse is transpose for orthonormal matrices
  - inverse is negative for translations

V2W vs. W2V

- $M_{W2V}=(M_{V2W})^{-1}=R^{-1}T^{-1}$

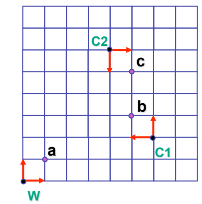
$$\mathbf{M}_{world2view} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u_x & u_y & u_z & -\mathbf{e} \bullet \mathbf{u} \\ v_x & v_y & v_z & -\mathbf{e} \bullet \mathbf{v} \\ w_x & w_y & w_z & -\mathbf{e} \bullet \mathbf{w} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{M}_{W2V} = \begin{bmatrix} u_x & u_y & u_z & -e_x * u_x + -e_y * u_y + -e_z * u_z \\ v_x & v_y & v_z & -e_x * v_x + -e_y * v_y + -e_z * v_z \\ w_x & w_y & w_z & -e_x * w_x + -e_y * w_y + -e_z * w_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Moving the Camera or the World?

- two equivalent operations
  - move camera one way vs. move world other way
- example
  - initial OpenGL camera: at origin, looking along -z axis
  - create a unit square parallel to camera at z = -10
  - translate in z by 3 possible in two ways
    - camera moves to z = -3
      - Note OpenGL models viewing in left-hand coordinates
    - camera stays put, but world moves to -7
  - resulting image same either way
    - possible difference: are lights specified in world or view coordinates?

World vs. Camera Coordinates Example



$\mathbf{a = (1,1)_W}$

$\mathbf{b = (1,1)_{C1} = (5,3)_W}$

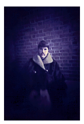
$\mathbf{c = (1,1)_{C2} = (1,3)_{C1} = (5,5)_W}$

Projections I

Pinhole Camera

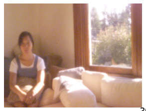
- ingredients
  - box, film, hole punch
- result
  - picture

www.kodak.com



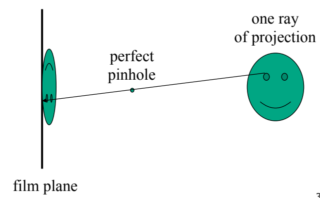
www.pinhole.org

www.debevec.org/Pinhole



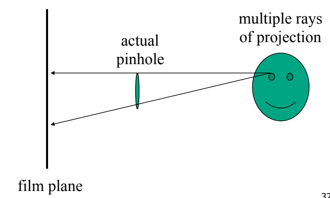
Pinhole Camera

- theoretical perfect pinhole
  - light shining through tiny hole into dark space yields upside-down picture



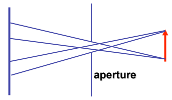
Pinhole Camera

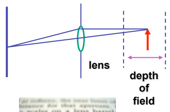
- non-zero sized hole
  - blur: rays hit multiple points on film plane

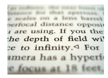


Real Cameras

- pinhole camera has small aperture (lens opening)
  - minimize blur
- problem: hard to get enough light to expose the film
- solution: lens
  - permits larger apertures
  - permits changing distance to film plane without actually moving it
    - cost: limited depth of field where image is in focus



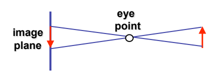




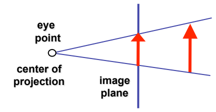
http://en.wikipedia.org/wiki/Image:DOF-ShallowDepthofField.jpg

Graphics Cameras

- real pinhole camera: image inverted

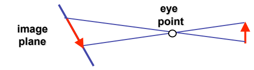


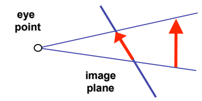
- computer graphics camera: convenient equivalent



General Projection

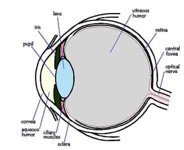
- image plane need not be perpendicular to view plane





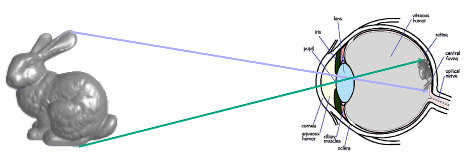
Perspective Projection

- our camera must model perspective



Perspective Projection

- our camera must model perspective



Projective Transformations

- planar geometric projections
  - planar: onto a plane
  - geometric: using straight lines
  - projections: 3D -> 2D
- aka projective mappings
- counterexamples?

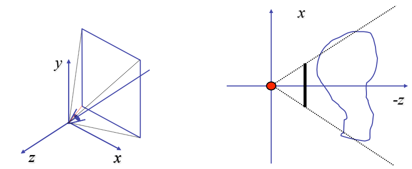
Projective Transformations

- properties
  - lines mapped to lines and triangles to triangles
  - parallel lines do NOT remain parallel
    - e.g. rails vanishing at infinity

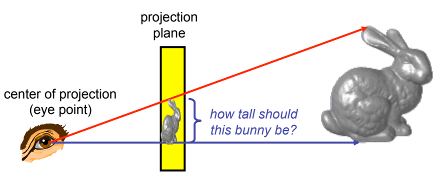
  - affine combinations are NOT preserved
    - e.g. center of a line does not map to center of projected line (perspective foreshortening)

Perspective Projection

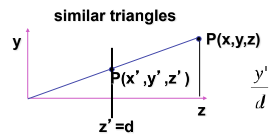
- project all geometry
  - through common center of projection (eye point)
  - onto an image plane



Perspective Projection



Basic Perspective Projection



$$\frac{y'}{d} = \frac{y}{z} \rightarrow y' = \frac{y \cdot d}{z}$$

$$\frac{x'}{d} = \frac{x}{z} \rightarrow x' = \frac{x \cdot d}{z} \quad \textbf{but} \quad z' = d$$

- nonuniform foreshortening
  - not affine

Perspective Projection

- desired result for a point $[x, y, z, 1]^T$ projected onto the view plane:

$$\frac{x'}{d} = \frac{x}{z}, \quad \frac{y'}{d} = \frac{y}{z}$$

$$x' = \frac{x \cdot d}{z} = \frac{x}{z/d}, \quad y' = \frac{y \cdot d}{z} = \frac{y}{z/d}, \quad z' = d$$

- what could a matrix look like to do this?

Simple Perspective Projection Matrix

$$\begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \end{bmatrix}$$

Simple Perspective Projection Matrix

$\begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \end{bmatrix}$ is homogenized version of $\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$

where w = z/d

Simple Perspective Projection Matrix

$\begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \end{bmatrix}$ is homogenized version of $\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$

where w = z/d

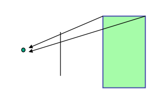
$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

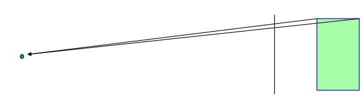
Perspective Projection

- expressible with 4x4 homogeneous matrix
  - use previously untouched bottom row
- perspective projection is irreversible
  - many 3D points can be mapped to same (x, y, d) on the projection plane
  - no way to retrieve the unique z values

Moving COP to Infinity

- as COP moves away, lines approach parallel
  - when COP at infinity, orthographic view





Orthographic Camera Projection

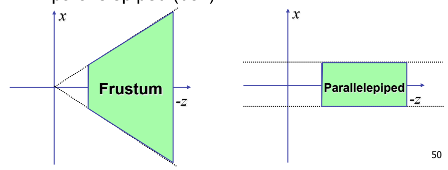
- camera's back plane parallel to lens
- infinite focal length
- no perspective convergence
- just throw away z values

$$\begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

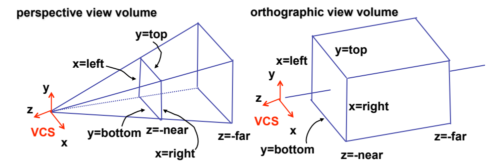
Perspective to Orthographic

- transformation of space
  - center of projection moves to infinity
  - view volume transformed
    - from frustum (truncated pyramid) to parallelepiped (box)



View Volumes

- specifies field-of-view, used for clipping
- restricts domain of **z** stored for visibility test

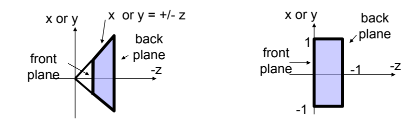


Canonical View Volumes

- standardized viewing volume representation

perspective

orthographic
orthogonal
parallel



Why Canonical View Volumes?

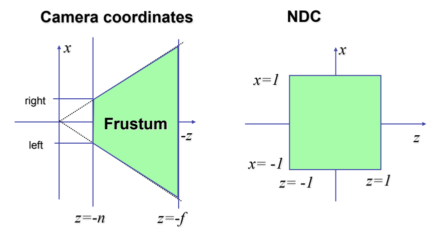
- permits standardization
  - clipping
    - easier to determine if an arbitrary point is enclosed in volume with canonical view volume vs. clipping to six arbitrary planes
  - rendering
    - projection and rasterization algorithms can be reused

Normalized Device Coordinates

- convention
  - viewing frustum mapped to specific parallelepiped
    - Normalized Device Coordinates (NDC)
    - same as clipping coords
  - only objects inside the parallelepiped get rendered
  - which parallelepiped?
    - depends on rendering system

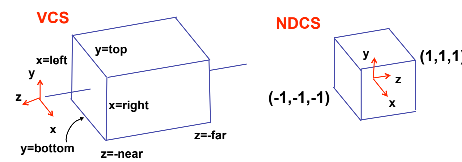
Normalized Device Coordinates

left/right x =+/- 1, top/bottom y =+/- 1, near/far z =+/- 1



Understanding Z

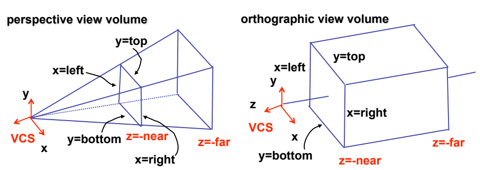
- z axis flip changes coord system handedness
  - RHS before projection (eye/view coords)
  - LHS after projection (clip, norm device coords)



Understanding Z

near, far always positive in OpenGL calls

```
glOrtho(left,right,bot,top,near,far);
glFrustum(left,right,bot,top,near,far);
glPerspective(fovy,aspect,near,far);
```

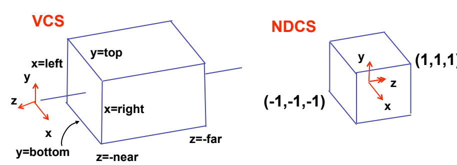


Understanding Z

- why near and far plane?
  - near plane:
    - avoid singularity (division by zero, or very small numbers)
  - far plane:
    - store depth in fixed-point representation (integer), thus have to have fixed range of values (0…1)
    - avoid/reduce numerical precision artifacts for distant objects

Orthographic Derivation

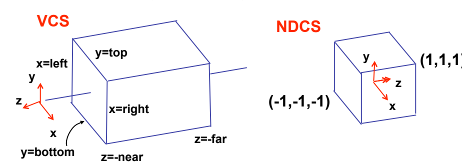
- scale, translate, reflect for new coord sys



Orthographic Derivation

- scale, translate, reflect for new coord sys

$$y' = a\cdot y + b \qquad \begin{array}{l} y = top \to y' = 1 \\ y = bot \to y' = -1 \end{array}$$



Orthographic Derivation

- scale, translate, reflect for new coord sys

$$y' = a\cdot y + b \qquad \begin{array}{ll} y = top \to y' = 1 & 1 = a\cdot top + b \\ y = bot \to y' = -1 & -1 = a\cdot bot + b \end{array}$$

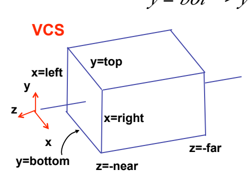
$$b = 1 - a\cdot top, b = -1 - a\cdot bot$$
$$1 - a\cdot top = -1 - a\cdot bot$$
$$1-(-1) = -a\cdot bot - (-a\cdot top)$$
$$2 = a(-bot + top)$$
$$a = \frac{2}{top - bot}$$

$$1 = \frac{2}{top - bot} top + b$$
$$b = 1 - \frac{2\cdot top}{top - bot}$$
$$b = \frac{(top - bot) - 2\cdot top}{top - bot}$$
$$b = \frac{-top - bot}{top - bot}$$

Orthographic Derivation

- scale, translate, reflect for new coord sys

$$y' = a\cdot y + b \qquad \begin{array}{l} y = top \to y' = 1 \\ y = bot \to y' = -1 \end{array}$$



$$a = \frac{2}{top - bot}$$

$$b = -\frac{top + bot}{top - bot}$$

same idea for right/left, far/near

Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P = \begin{bmatrix} \frac{2}{right - left} & 0 & 0 & -\frac{right + left}{right - left} \\ 0 & \frac{2}{top - bot} & 0 & -\frac{top + bot}{top - bot} \\ 0 & 0 & \frac{-2}{far - near} & -\frac{far + near}{far - near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P = \begin{bmatrix} \boxed{\frac{2}{right - left}} & 0 & 0 & -\frac{right + left}{right - left} \\ 0 & \boxed{\frac{2}{top - bot}} & 0 & -\frac{top + bot}{top - bot} \\ 0 & 0 & \boxed{\frac{-2}{far - near}} & -\frac{far + near}{far - near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bot} & 0 & -\frac{top+bot}{top-bot} \\ 0 & 0 & \frac{-2}{far-near} & -\frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bot} & 0 & -\frac{top+bot}{top-bot} \\ 0 & 0 & \frac{-2}{far-near} & -\frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic OpenGL

```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
glOrtho(left,right,bot,top,near,far);
```

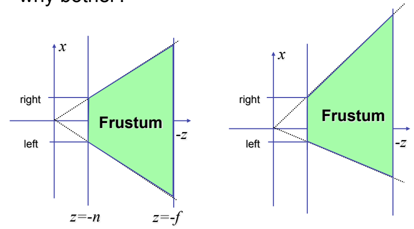

Demo

- Brown applets: viewing techniques
  - parallel/orthographic cameras
  - projection cameras
- http://www.cs.brown.edu/exploratories/freeSoftware/catalogs/viewing_techniques.html

Projections II

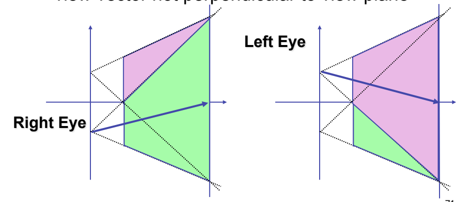
Asymmetric Frusta

- our formulation allows asymmetry
  - why bother?



Asymmetric Frusta

- our formulation allows asymmetry
  - why bother? binocular stereo
    - view vector not perpendicular to view plane

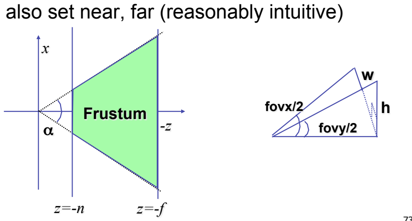


Simpler Formulation

- left, right, bottom, top, near, far
  - nonintuitive
  - often overkill
- look through window center
  - symmetric frustum
- constraints
  - left = -right, bottom = -top

Field-of-View Formulation

- FOV in one direction + aspect ratio (w/h)
  - determines FOV in other direction
  - also set near, far (reasonably intuitive)



Perspective OpenGL

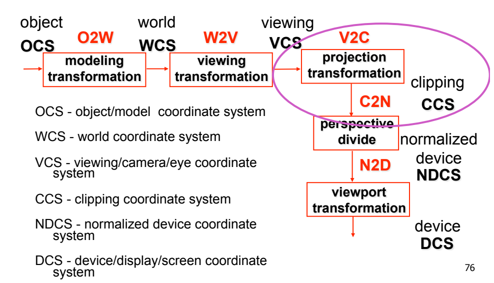
```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();

glFrustum(left,right,bot,top,near,far);
 or
glPerspective(fovy,aspect,near,far);
```


Demo: Frustum vs. FOV

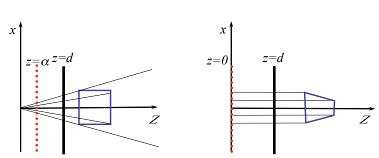
- Nate Robins tutorial (take 2):
  - http://www.xmission.com/~nate/tutors.html

Projective Rendering Pipeline



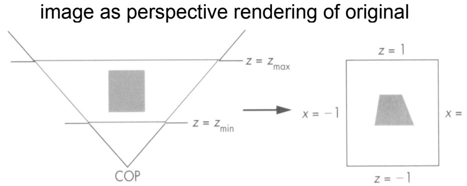
Projection Warp

- warp perspective view volume to orthogonal view volume
  - render all scenes with orthographic projection!
  - aka perspective warp

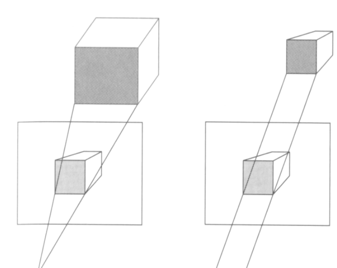


Perspective Warp

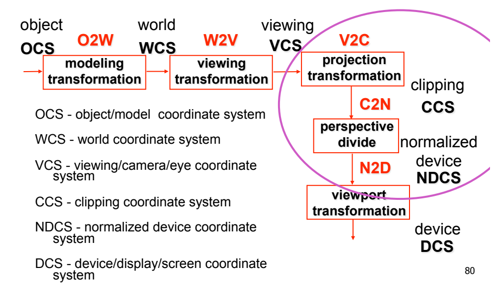
- perspective viewing frustum transformed to cube
- orthographic rendering of cube produces same image as perspective rendering of original



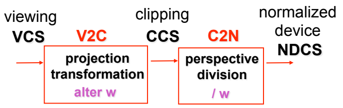
Predistortion



Projective Rendering Pipeline



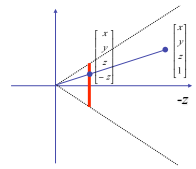
Separate Warp From Homogenization



- warp requires only standard matrix multiply
  - distort such that orthographic projection of distorted objects is desired persp projection
    - w is changed
  - clip after warp, before divide
  - division by w: homogenization

Perspective Divide Example

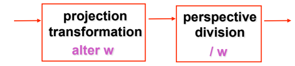
- specific example
- assume image plane at $z = -1$
- a point $[x,y,z,1]^T$ projects to $[-x/z,-y/z,-z/z,1]^T \equiv [x,y,z,-z]^T$



Perspective Divide Example

$$T\left(\begin{bmatrix}x\\y\\z\\1\end{bmatrix}\right)=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&-1&0\end{bmatrix}\cdot\begin{bmatrix}x\\y\\z\\1\end{bmatrix}=\begin{bmatrix}x\\y\\z\\-z\end{bmatrix}\equiv\begin{bmatrix}-x/z\\-y/z\\-1\\1\end{bmatrix}$$

- after homogenizing, once again w=1



Perspective Normalization

- matrix formulation

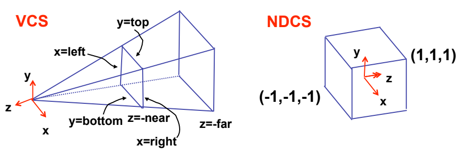
$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\frac{d}{d-\alpha}&\frac{-\alpha\cdot d}{d-\alpha}\\0&0&\frac{1}{d}&0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}=\begin{bmatrix}x\\y\\\left(\frac{(z-\alpha)\cdot d}{d-\alpha}\right)\\\frac{z}{d}\end{bmatrix}\qquad\begin{bmatrix}x_p\\y_p\\z_p\end{bmatrix}=\begin{bmatrix}\frac{x}{z/d}\\\frac{y}{z/d}\\\frac{d^2}{d-\alpha}\left(1-\frac{\alpha}{z}\right)\end{bmatrix}$$

- warp and homogenization both preserve relative depth (z coordinate)

Demo

- Brown applets: viewing techniques
  - parallel/orthographic cameras
  - projection cameras
- http://www.cs.brown.edu/exploratories/freeSoftware/catalogs/viewing_techniques.html

Perspective To NDCS Derivation



Perspective Derivation

simple example earlier:

$$\begin{bmatrix}x'\\y'\\z'\\w'\end{bmatrix}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/d&0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

complete: shear, scale, projection-normalization

$$\begin{bmatrix}x'\\y'\\z'\\w'\end{bmatrix}=\begin{bmatrix}E&0&A&0\\0&F&B&0\\0&0&C&D\\0&0&-1&0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Perspective Derivation

earlier:

$$\begin{bmatrix}x'\\y'\\z'\\w'\end{bmatrix}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/d&0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

complete: shear, scale, projection-normalization

$$\begin{bmatrix}x'\\y'\\z'\\w'\end{bmatrix}=\begin{bmatrix}E&0&A&0\\0&F&B&0\\0&0&C&D\\0&0&-1&0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Perspective Derivation

earlier:

$$\begin{bmatrix}x'\\y'\\z'\\w'\end{bmatrix}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/d&0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

complete: shear, scale, projection-normalization

$$\begin{bmatrix}x'\\y'\\z'\\w'\end{bmatrix}=\begin{bmatrix}E&0&A&0\\0&F&B&0\\0&0&C&D\\0&0&-1&0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Recorrection: Perspective Derivation

$$\begin{bmatrix}x'\\y'\\z'\\w'\end{bmatrix}=\begin{bmatrix}E&0&A&0\\0&F&B&0\\0&0&C&D\\0&0&-1&0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

$x' = Ex + Az$
$y' = Fy + Bz$
$z' = Cz + D$
$w' = -z$

L/R sign error

$x = left \rightarrow x'/w' = -1$
$x = right \rightarrow x'/w' = 1$
$y = top \rightarrow y'/w' = 1$
$y = bottom \rightarrow y'/w' = -1$
$z = -near \rightarrow z'/w' = -1$
$z = -far \rightarrow z'/w' = 1$

z axis flip!

$$y' = Fy + Bz,\quad \frac{y'}{w'} = \frac{Fy+Bz}{w'},\quad 1 = \frac{Fy+Bz}{w'},\quad 1 = \frac{Fy+Bz}{-z},$$

$$1 = F\frac{y}{-z} + B\frac{z}{-z},\quad 1 = F\frac{y}{-z} - B,\quad 1 = F\frac{top}{-(-near)} - B,$$

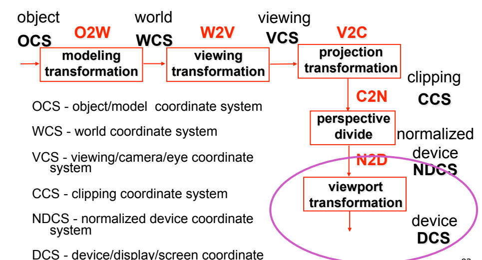
$$1 = F\frac{top}{near} - B$$

Perspective Derivation

- similarly for other 5 planes
- 6 planes, 6 unknowns

$$\begin{bmatrix}\frac{2n}{r-l}&0&\frac{r+l}{r-l}&0\\0&\frac{2n}{t-b}&\frac{t+b}{t-b}&0\\0&0&\frac{-(f+n)}{f-n}&\frac{-2fn}{f-n}\\0&0&-1&0\end{bmatrix}$$

Projective Rendering Pipeline



OCS - object/model coordinate system

WCS - world coordinate system

VCS - viewing/camera/eye coordinate system

CCS - clipping coordinate system

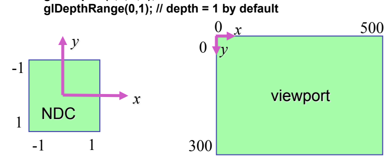
NDCS - normalized device coordinate system

DCS - device/display/screen coordinate system

NDC to Device Transformation

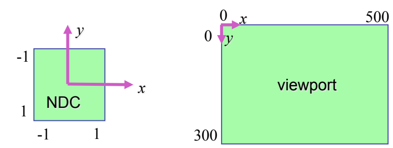
- map from NDC to pixel coordinates on display
  - NDC range is x = -1...1, y = -1...1, z = -1...1
  - typical display range: x = 0...500, y = 0...300
    - maximum is size of actual screen
    - z range max and default is (0, 1), use later for visibility

```
glViewport(0,0,w,h);
glDepthRange(0,1); // depth = 1 by default
```



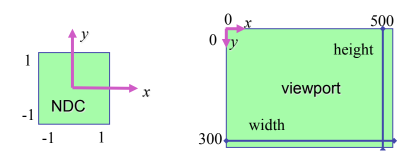
Origin Location

- yet more (possibly confusing) conventions
  - OpenGL origin: lower left
  - most window systems origin: upper left
- then must reflect in y
- when interpreting mouse position, have to flip your y coordinates



N2D Transformation

- general formulation
  - reflect in y for upper vs. lower left origin
  - scale by width, height, depth
  - translate by width/2, height/2, depth/2
    - FCG includes additional translation for pixel centers at (.5, .5) instead of (0,0)

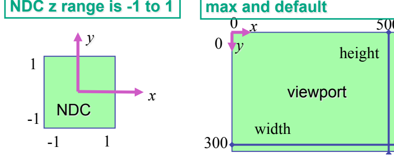


N2D Transformation

$$\begin{bmatrix}x_D\\y_D\\z_D\\1\end{bmatrix}=\begin{bmatrix}1&0&0&\frac{width}{2}-\frac{1}{2}\\0&1&0&\frac{height}{2}-\frac{1}{2}\\0&0&1&\frac{depth}{2}\\0&0&0&1\end{bmatrix}\begin{bmatrix}\frac{width}{2}&0&0&0\\0&\frac{height}{2}&0&0\\0&0&\frac{depth}{2}&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}x_N\\y_N\\z_N\\1\end{bmatrix}=\begin{bmatrix}\frac{width(x_N+1)-1}{2}\\\frac{height(-y_N+1)-1}{2}\\\frac{depth(z_N+1)}{2}\\1\end{bmatrix}$$

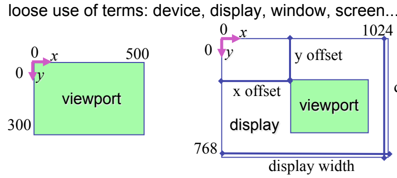
reminder:
NDC z range is -1 to 1

Display z range is 0 to 1. glDepthRange(n,f) can constrain further, but *depth* = 1 is both max and default

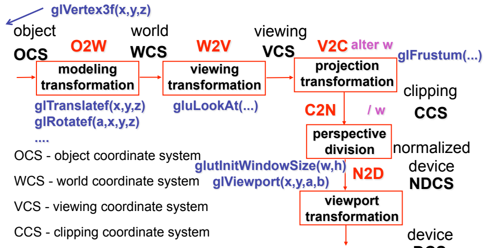


Device vs. Screen Coordinates

- viewport/window location wrt actual display not available within OpenGL
  - usually don't care
    - use relative information when handling mouse events, not absolute coordinates
  - could get actual display height/width, window offsets from OS
- loose use of terms: device, display, window, screen...



Projective Rendering Pipeline



OCS - object coordinate system

WCS - world coordinate system

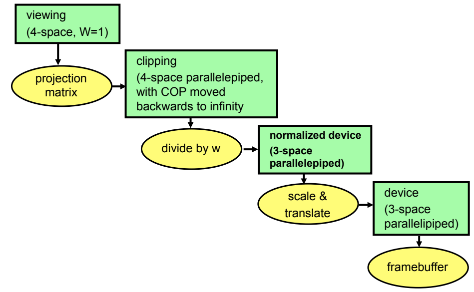
VCS - viewing coordinate system

CCS - clipping coordinate system

NDCS - normalized device coordinate system

DCS - device coordinate system

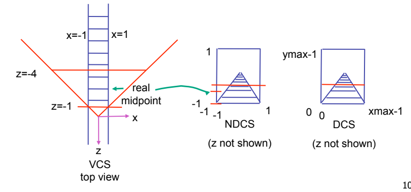
Coordinate Systems



Perspective Example

tracks in VCS:
left x=-1, y=-1
right x=1, y=-1

view volume
left = -1, right = 1
bot = -1, top = 1
near = 1, far = 4



Perspective Example

view volume

- left = -1, right = 1
- bot = -1, top = 1
- near = 1, far = 4

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5/3 & -8/3 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

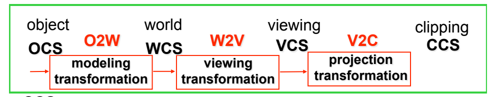
Perspective Example

$$\begin{bmatrix} 1 \\ -1 \\ -5z_{VCS}/3 - 8/3 \\ -z_{VCS} \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -5/3 & -8/3 \\ & & -1 & \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ z_{VCS} \\ 1 \end{bmatrix}$$

/ w

$$x_{NDCS} = -1/z_{VCS}$$
$$y_{NDCS} = 1/z_{VCS}$$
$$z_{NDCS} = \frac{5}{3} + \frac{8}{3z_{VCS}}$$

OpenGL Example




```
CCS   glMatrixMode( GL_PROJECTION );
      glLoadIdentity();
      gluPerspective( 45, 1.0, 0.1, 200.0 );
VCS   glMatrixMode( GL_MODELVIEW );
      glLoadIdentity();
      glTranslatef( 0.0, 0.0, -5.0 );
WCS   glPushMatrix()
      glTranslate( 4, 4, 0 );   W2O
OCS1  glutSolidTeapot(1);
      glPopMatrix();
      glTranslate( 2, 2, 0);    W2O
OCS2  glutSolidTeapot(1);
```

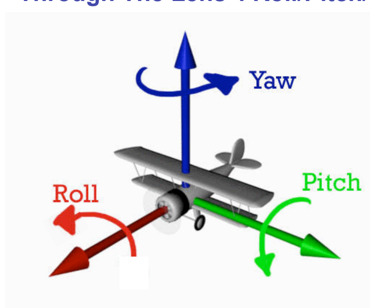
- transformations that are applied to object first are specified last

Reading for Next Time

- RB Chap Color
- FCG Sections 3.2-3.3
- FCG Chap 20 Color
- FCG Chap 21.2.2 Visual Perception (Color)

Viewing: More Camera Motion

Fly "Through The Lens": Roll/Pitch/Yaw



Viewing: Incremental Relative Motion

- how to move relative to current camera coordinate system?
  - what you see in the window
- computation in coordinate system used to draw previous frame is simple:
  - incremental change I to current C
  - at time k, want p' = $I_k I_{k-1} I_{k-2} I_{k-3} \ldots I_5 I_4 I_3 I_2 I_1 Cp$
- each time we just want to premultiply by new matrix
  - p'=ICp
  - but we know that OpenGL only supports postmultiply by new matrix
    - p'=CIp

Viewing: Incremental Relative Motion

- sneaky trick: OpenGL modelview matrix has the info we want!
  - dump out modelview matrix from previous frame with glGetDoublev()
    - C = current camera coordinate matrix
  - wipe the matrix stack with glIdentity()
  - apply incremental update matrix I
  - apply current camera coord matrix C
- must leave the modelview matrix unchanged by object transformations after your display call
  - use push/pop
- using OpenGL for storage and calculation
  - querying pipeline is expensive
    - but safe to do just once per frame

Caution: OpenGL Matrix Storage

- OpenGL internal matrix storage is columnwise, not rowwise

```
a  e  i  m
b  f  j  n
c  g  k  o
d  h  l  p
```

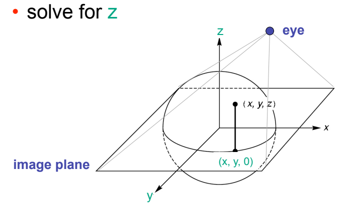
  - opposite of standard C/C++/Java convention
  - possibly confusing if you look at the matrix from glGetDoublev()!

Viewing: Virtual Trackball

- interface for spinning objects around
  - drag mouse to control rotation of view volume
    - orbit/spin metaphor
    - vs. flying/driving
- rolling glass trackball
  - center at screen origin, surrounds world
  - hemisphere "sticks up" in z, out of screen
  - rotate ball = spin world

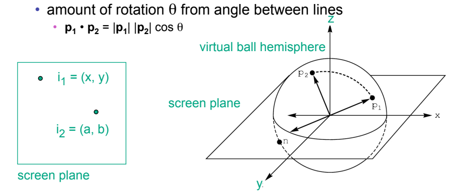
Clarify: Virtual Trackball

- know screen click: (x, y, 0)
- want to infer point on trackball: (x,y,z)
  - ball is unit sphere, so ||x, y, z|| = 1.0
  - solve for z



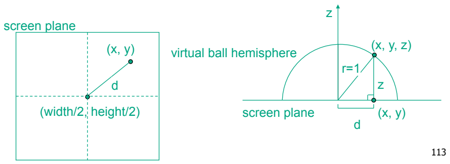
Clarify: Trackball Rotation

- user drags between two points on image plane
  - mouse down at i_1 = (x, y), mouse up at i_2 = (a, b)
- find corresponding points on virtual ball
  - p_1 = (x, y, z), p_2 = (a, b, c)
- compute rotation angle and axis for ball
  - axis of rotation is plane normal: cross product $p_1 \times p_2$
  - amount of rotation θ from angle between lines
    - $p_1 \cdot p_2 = |p_1|\,|p_2| \cos\theta$



Clarify: Trackball Rotation

- finding location on ball corresponding to click on image plane
  - ball radius r is 1



Trackball Computation

- user defines two points
  - place where first clicked $\mathbf{p_1}$ = (x, y, z)
  - place where released $\mathbf{p_2}$ = (a, b, c)
- create plane from vectors between points, origin
  - axis of rotation is plane normal: cross product
    - $(\mathbf{p_1} - \mathbf{o}) \times (\mathbf{p_2} - \mathbf{o})$: $\mathbf{p_1} \times \mathbf{p_2}$ if origin = (0,0,0)
  - amount of rotation depends on angle between lines
    - $\mathbf{p_1} \bullet \mathbf{p_2} = |\mathbf{p_1}|\,|\mathbf{p_2}| \cos\theta$
    - $|\mathbf{p_1} \times \mathbf{p_2}| = |\mathbf{p_1}|\,|\mathbf{p_2}| \sin\theta$
- compute rotation matrix, use to rotate world

Picking

Reading

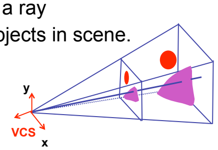
- Red Book
  - Selection and Feedback Chapter
    - all
  - Now That You Know Chapter
    - only Object Selection Using the Back Buffer

Interactive Object Selection

- move cursor over object, click
  - how to decide what is below?
  - inverse of rendering pipeline flow
    - from pixel back up to object
- ambiguity
  - many 3D world objects map to same 2D point
- four common approaches
  - manual ray intersection
  - bounding extents
  - backbuffer color coding
  - selection region with hit list

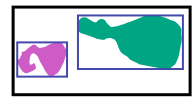
Manual Ray Intersection

- do all computation at application level
  - map selection point to a ray
  - intersect ray with all objects in scene.
- advantages
  - no library dependence
- disadvantages
  - difficult to program
  - slow: work to do depends on total number and complexity of objects in scene



Bounding Extents

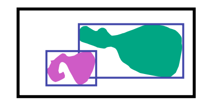
- keep track of axis-aligned bounding rectangles



- advantages
  - conceptually simple
  - easy to keep track of boxes in world space

Bounding Extents

- disadvantages
  - low precision
  - must keep track of object-rectangle relationship
- extensions
  - do more sophisticated bound bookkeeping
    - first level: box check.
    - second level: object check

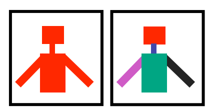


Backbuffer Color Coding

- use backbuffer for picking
  - create image as computational entity
  - never displayed to user
- redraw all objects in backbuffer
  - turn off shading calculations
  - set unique color for each pickable object
    - store in table
  - read back pixel at cursor location
    - check against table

Backbuffer Color Coding

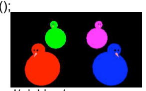
- advantages
  - conceptually simple
  - variable precision
- disadvantages
  - introduce 2x redraw delay
  - backbuffer readback very slow



Backbuffer Example

```
glColor3f(1.0, 1.0, 1.0);
for(int i = 0; i < 2; i++)
    for(int j = 0; j < 2; j++) {
        glPushMatrix();
        glTranslatef(i*3.0,0,-j * 3.0);
        glColor3f(1.0, 1.0, 1.0);
        glCallList(snowman_display_list);
        glPopMatrix();
  }
```

```
for(int i = 0; i < 2; i++)
  for(int j = 0; j < 2; j++) {
     glPushMatrix();
     switch (i*2+j) {
        case 0: glColor3ub(255,0,0);break;
        case 1: glColor3ub(0,255,0);break;
        case 2: glColor3ub(0,0,255);break;
        case 3: glColor3ub(250,0,250);break;
     }
     glTranslatef(i*3.0,0,-j * 3.0)
     glCallList(snowman_display_list);
     glPopMatrix();
  }
```



http://www.lighthouse3d.com/opengl/picking/

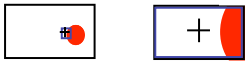
Select/Hit

- use small region around cursor for viewport
- assign per-object integer keys (names)
- redraw in special mode
- store hit list of objects in region
- examine hit list

- OpenGL support

Viewport

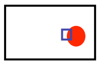
- small rectangle around cursor
  - change coord sys so fills viewport



- why rectangle instead of point?
  - people aren't great at positioning mouse
    - Fitts' Law: time to acquire a target is function of the distance to and size of the target
  - allow several pixels of slop

Viewport

- nontrivial to compute
  - invert viewport matrix, set up new orthogonal projection
- simple utility command
  - gluPickMatrix(x,y,w,h,viewport)
    - x,y: cursor point
    - w,h: sensitivity/slop (in pixels)
  - push old setup first, so can pop it later



Render Modes

- glRenderMode(mode)
  - GL_RENDER: normal color buffer
    - default
  - GL_SELECT: selection mode for picking
  - (GL_FEEDBACK: report objects drawn)

Name Stack

- again, "names" are just integers
  glInitNames()
- flat list
  glLoadName(name)
- or hierarchy supported by stack
  glPushName(name), glPopName
  - can have multiple names per object

Hierarchical Names Example

```
for(int i = 0; i < 2; i++) {
  glPushName(i);
  for(int j = 0; j < 2; j++) {
    glPushMatrix();
    glPushName(j);
    glTranslatef(i*10.0,0,j * 10.0);
      glPushName(HEAD);
      glCallList(snowManHeadDL);
      glLoadName(BODY);
      glCallList(snowManBodyDL);
      glPopName();
    glPopName();
    glPopMatrix();
  }
  glPopName();
}
```

http://www.lighthouse3d.com/opengl/picking/

Hit List

- glSelectBuffer(buffersize, *buffer)
  - where to store hit list data
- on hit, copy entire contents of name stack to output buffer.
- hit record
  - number of names on stack
  - minimum and maximum depth of object vertices
    - depth lies in the NDC z range [0,1]
    - format: multiplied by 2^32 -1 then rounded to nearest int

Integrated vs. Separate Pick Function

- integrate: use same function to draw and pick
  - simpler to code
  - name stack commands ignored in render mode
- separate: customize functions for each
  - potentially more efficient
  - can avoid drawing unpickable objects

Select/Hit

- advantages
  - faster
    - OpenGL support means hardware acceleration
    - avoid shading overhead
  - flexible precision
    - size of region controllable
  - flexible architecture
    - custom code possible, e.g. guaranteed frame rate
- disadvantages
  - more complex

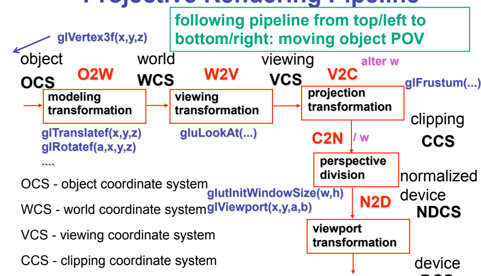
Hybrid Picking

- select/hit approach: fast, coarse
  - object-level granularity
- manual ray intersection: slow, precise
  - exact intersection point
- hybrid: both speed and precision
  - use select/hit to find object
  - then intersect ray with that object

OpenGL Precision Picking Hints

- gluUnproject
  - transform window coordinates to object coordinates given current projection and modelview matrices
  - use to create ray into scene from cursor location
  - call gluUnProject twice with same (x,y) mouse location
    - z = near: (x,y,0)
    - z = far: (x,y,1)
    - subtract near result from far result to get direction vector for ray
- use this ray for line/polygon intersection

Projective Rendering Pipeline



OCS - object coordinate system

WCS - world coordinate system

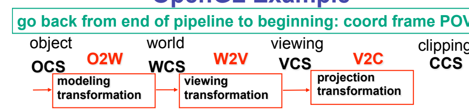
VCS - viewing coordinate system

CCS - clipping coordinate system

NDCS - normalized device coordinate system

DCS - device coordinate system

OpenGL Example



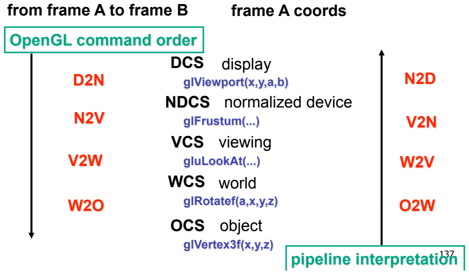

```
CCS   glMatrixMode( GL_PROJECTION );
      glLoadIdentity();
      gluPerspective( 45, 1.0, 0.1, 200.0 );
VCS   glMatrixMode( GL_MODELVIEW );
      glLoadIdentity();
      glTranslatef( 0.0, 0.0, -5.0 );        V2W
WCS   glPushMatrix()
      glTranslate( 4, 4, 0 );   W2O
OCS1  glutSolidTeapot(1);
      glPopMatrix();
      glTranslate( 2, 2, 0);    W2O
OCS2  glutSolidTeapot(1);
```

- transformations that are applied to object first are specified last

Coord Sys: Frame vs Point

read down: transforming between coordinate frames, from frame A to frame B

read up: transforming points, up from frame B coords to frame A coords



Coord Sys: Frame vs Point

- is gluLookat viewing transformation V2W or W2V? depends on which way you read!
  - coordinate frames: V2W
    - takes you from view to world coordinate frame
  - points/objects: W2V
    - point is transformed from world to view coords when multiply by gluLookAt matrix
- H2 uses the object/pipeline POV
  - Q1/4 is W2V (gluLookAt)
  - Q2/5-6 is V2N (glFrustum)
  - Q3/7 is N2D (glViewport)