University of British Columbia
CPSC 314 Computer Graphics
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Tamara Munzner

Rasterization

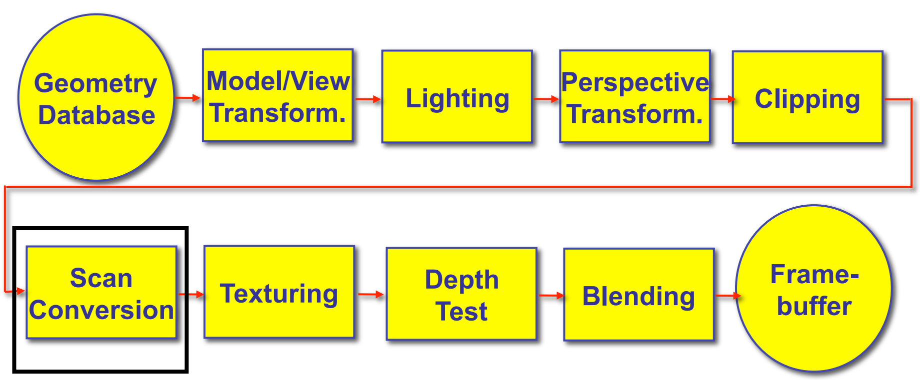
http://www.ugrad.cs.ubc.ca/~cs314/Vjan2013

Reading for This Module

- FCG Chap 3 Raster Algorithms (through 3.2)
- Section 2.7 Triangles
- Section 8.1 Rasterization (through 8.1.2)

Rasterization

Rendering Pipeline

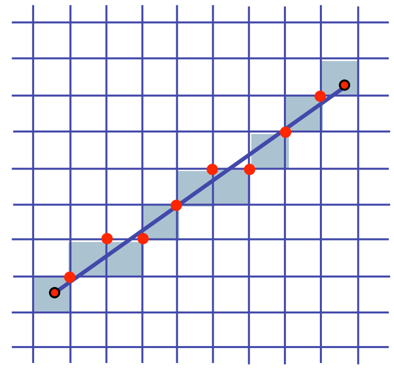
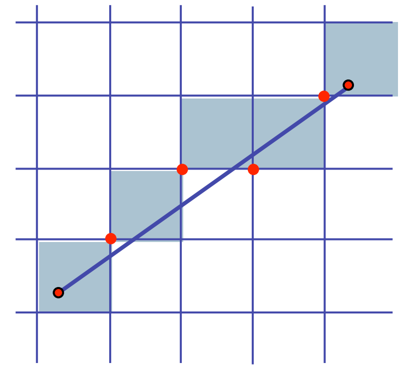


Scan Conversion - Rasterization

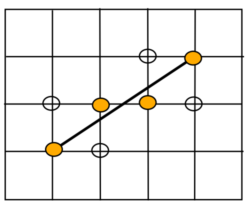
- convert continuous rendering primitives into discrete fragments/pixels
  - lines
    - midpoint/Bresenham
  - triangles
    - flood fill
    - scanline
    - implicit formulation
  - interpolation

Scan Conversion

- given vertices in DCS, fill in the pixels
- display coordinates required to provide scale for discretization
  - [demo]

Basic Line Drawing



$$y = mx + b$$

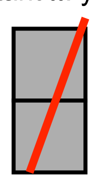
$$y = \frac{(y_1 - y_0)}{(x_1 - x_0)}(x - x_0) + y_0$$

- goals
  - integer coordinates
  - thinnest line with no gaps
- assume
  - $x_0 < x_1$, slope $0 < {}^{dy}\!/_{dx} < 1$
  - one octant, other cases symmetric
- how can we do this more quickly?

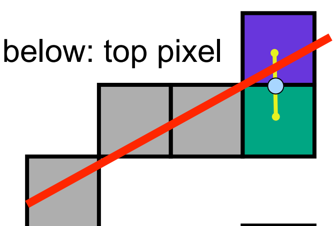
Line (x_0, y_0, x_1, y_1)
begin
float $dx, dy, x, y, slope$;
$dx \Leftarrow x_1 - x_0$;
$dy \Leftarrow y_1 - y_0$;
$slope \Leftarrow {}^{dy}\!/_{dx}$;
$y \Leftarrow y_0$
for x from x_0 to x_1 do
  begin
    PlotPixel (x, **Round** (y)) ;
    $y \Leftarrow y + slope$;
  end ;
end ;

Midpoint Algorithm

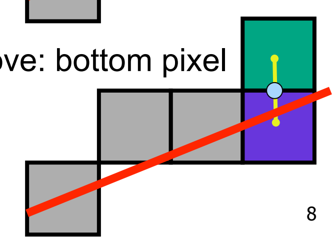
- we're moving horizontally along x direction (first octant)
  - only two choices: draw at current y value, or move up vertically to y +1?
    - check if midpoint between two possible pixel centers above or below line
  - candidates
    - top pixel: (x+1,y+1)
    - bottom pixel: (x+1, y)
  - midpoint: (x+1, y+.5)
- check if midpoint above or below line
  - below: pick top pixel
  - above: pick bottom pixel
- other octants: different tests
  - octant II: y loop, check x left/right



below: top pixel

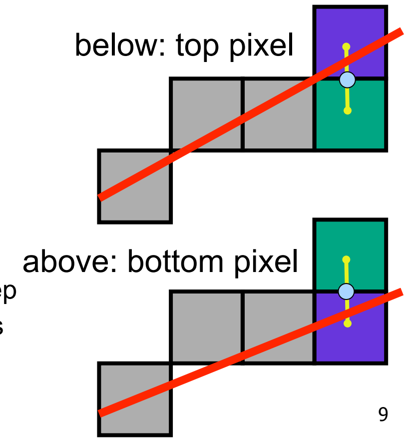


above: bottom pixel



Midpoint Algorithm

- we're moving horizontally along x direction (first octant)
  - only two choices: draw at current y value, or move up vertically to y +1?
    - check if midpoint between two possible pixel centers above or below line
  - candidates
    - top pixel: (x+1,y+1)
    - bottom pixel: (x+1, y)
  - midpoint: (x+1, y+.5)
- check if midpoint above or below line
  - below: pick top pixel
  - above: pick bottom pixel
- key idea behind Bresenham
  - reuse computation from previous step
  - integer arithmetic by doubling values
  - [demo]

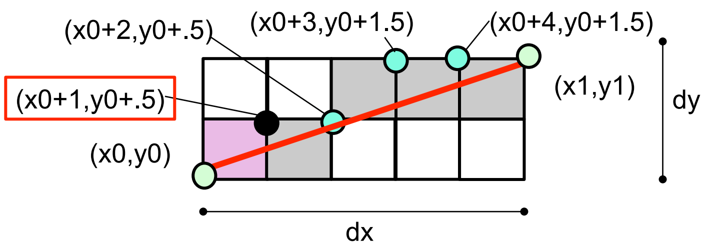


Bresenham, Detailed Derivation

- Goal: function F tells us if line is above or below some point
  - F(x,y) = 0 on line
  - F(x,y) < 0 when line under point
  - F(x,y) > 0 when line over point

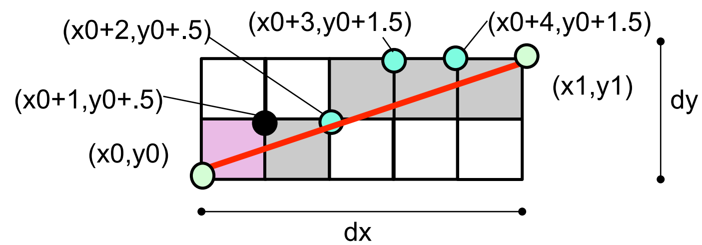
$$y = mx + b$$
$$y = \frac{dy}{dx}x + b$$
$$dx * y = dy * x + b * dx$$
$$0 = dy * x - dx * y + b * dx$$
$$2 * 0 = 2 * dy * x - 2 * dx * y + 2 * b * dx$$
$$0 = 2 * dy * x - 2 * dx * y + 2 * b * dx$$
$$F(x,y) = 2 * dy * x - 2 * dx * y + 2 * b * dx$$

Using F with Midpoints: Initial



$$F(x_0,y_0) = 2 * dy * x_0 - 2 * dx * y_0 + 2 * b * dx$$
$$F(x_0 + 1, y_0 + .5)$$
$$= 2 * dy * (x_0 + 1) - 2 * dx * (y_0 + .5) + 2 * b * dx$$
$$= 2 * dy * x_0 + 2 * dy - 2 * dx * y_0 - dx + 2 * b * dx$$

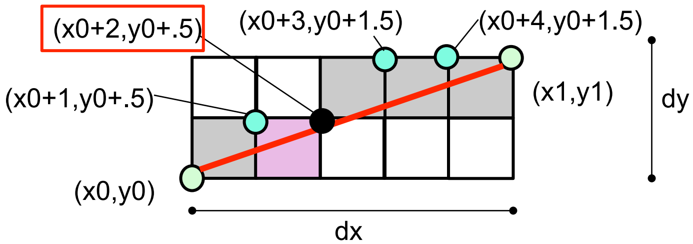
Incremental F: Initial



$$F(x_0,y_0) = \boxed{2 * dy * x_0} - \boxed{2 * dx * y_0} + \boxed{2 * b * dx}$$
$$F(x_0 + 1, y_0 + .5)$$
$$= 2 * dy * (x_0 + 1) - 2 * dx * (y_0 + .5) + 2 * b * dx$$
$$= \boxed{2 * dy * x_0} + \underline{2 * dy} - \boxed{2 * dx * y_0} - \underline{dx} + \boxed{2 * b * dx}$$
$$F(x_0 + 1, y_0 + .5) - F(x_0, y_0) = \underline{2 * dy - dx} = \mathbf{diff}$$

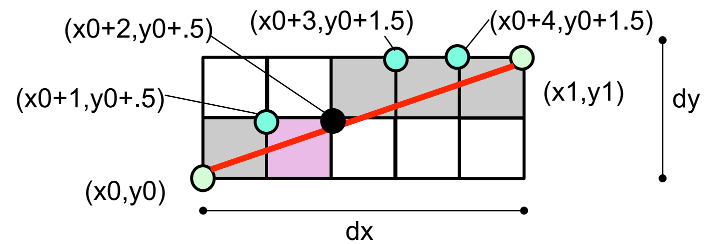
- Initial difference in F: 2*dy-dx

Using F with Midpoints: No Y Change



$$F(x_0+1, y_0+.5)$$
$$= 2*dy*x_0 + 2*dy - 2*dx*y_0 - dx + 2*b*dx$$
$$F(x_0+2, y_0+.5)$$
$$= 2*dy*(x_0+2) - 2*dx*(y_0+.5) + 2*b*dx$$
$$= 2*dy*x_0 + 4*dy - 2*dx*y_0 - dx + 2*b*dx$$

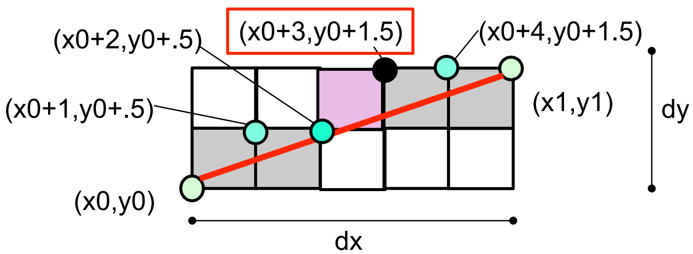
Incremental F: No Y Change



$$F(x_0+1, y_0+.5)$$
$$= \boxed{2*dy*x_0} + \underline{2*dy} - \boxed{2*dx*y_0} - \boxed{dx + 2*b*dx}$$
$$F(x_0+2, y_0+.5)$$
$$= 2*dy*(x_0+2) - 2*dx*(y_0+.5) + 2*b*dx$$
$$= \boxed{2*dy*x_0} + \underline{4*dy} - \boxed{2*dx*y_0} - \boxed{dx + 2*b*dx}$$
$$F(x_0+2, y_0+.5) - F(x_0+1, y_0+.5) = \underline{2*dy} = \mathbf{diff}$$

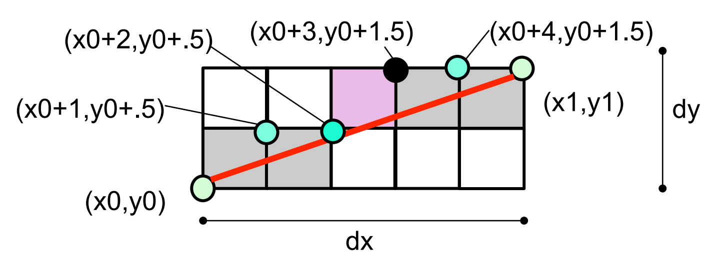
- Next difference in F: 2*dy (no change in y for pixel)

Using F with Midpoints: Y Increased



$$F(x_0+2, y_0+.5)$$
$$= 2*dy*x_0 + 4*dy - 2*dx*y_0 - dx + 2*b*dx$$
$$F(x_0+3, y_0+1.5)$$
$$= 2*dy*(x_0+3) - 2*dx*(y_0+1.5) + 2*b*dx$$
$$= 2*dy*x_0 + 6*dy - 2*dx*y_0 - 3*dx + 2*b*dx$$

Incremental F: Y Increased



$$F(x_0+2, y_0+.5)$$
$$= \boxed{2*dy*x_0} + \underline{4*dy} - \boxed{2*dx*y_0} - \underline{dx} + \boxed{2*b*dx}$$
$$F(x_0+3, y_0+1.5)$$
$$= 2*dy*(x_0+3) - 2*dx*(y_0+1.5) + 2*b*dx$$
$$= \boxed{2*dy*x_0} + \underline{6*dy} - \boxed{2*dx*y_0} - \underline{3*dx} + \boxed{2*b*dx}$$
$$F(x_0+3, y_0+1.5) - F(x_0+2, y_0+.5) = \underline{2*dy - 2*dx} = \mathbf{diff}$$

- Next difference in F: 2*dy-2*dx (when pixel at y+1)

Bresenham:
Reuse Computation, Integer Only

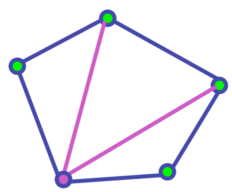
```
y=y0;
dx = x1-x0;
dy = y1-y0;
d = 2*dy-dx;
incKeepY = 2*dy;
incIncreaseY = 2*dy-2*dx;
for (x=x0; x <= x1; x++) {
  draw(x,y);
  if (d>0) then {
   y = y + 1;
   d += incIncreaseY;
  } else {
   d += incKeepY;
}
```


Rasterizing Polygons/Triangles

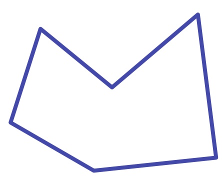
- basic surface representation in rendering
- why?
  - lowest common denominator
    - can approximate any surface with arbitrary accuracy
      - all polygons can be broken up into triangles
  - guaranteed to be:
    - planar
    - triangles - convex
  - simple to render
    - can implement in hardware

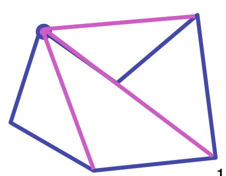
Triangulating Polygons

- simple convex polygons
  - trivial to break into triangles
  - pick one vertex, draw lines to all others not immediately adjacent
  - OpenGL supports automatically
    - glBegin(GL_POLYGON) ... glEnd()



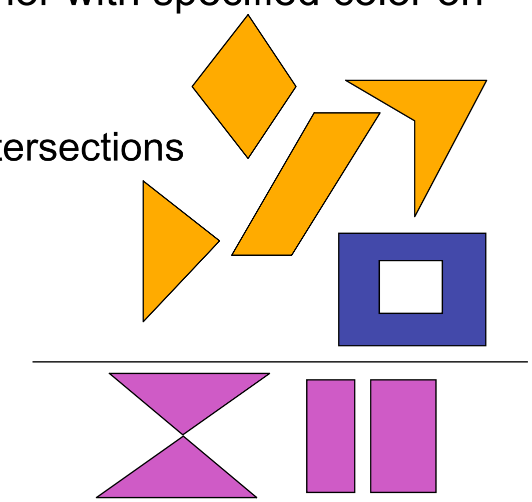
- concave or non-simple polygons
  - more effort to break into triangles
  - simple approach may not work
  - OpenGL can support at extra cost
    - gluNewTess(), gluTessCallback(), ...





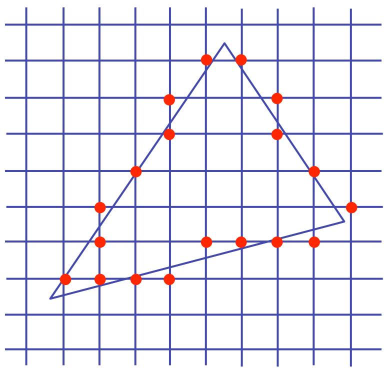
Problem

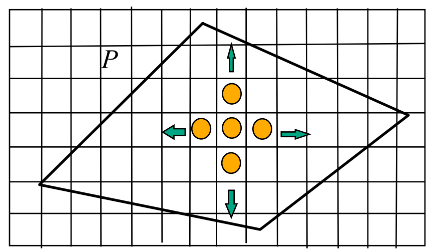
- input: closed 2D polygon
- problem: fill its interior with specified color on graphics display
- assumptions
  - simple - no self intersections
  - simply connected
- solutions
  - flood fill
  - edge walking



Flood Fill

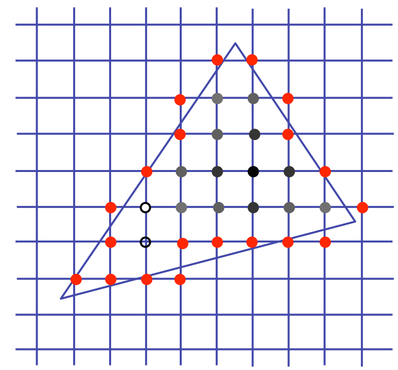
- simple algorithm
  - draw edges of polygon
  - use flood-fill to draw interior





Flood Fill

- start with seed point
- recursively set all neighbors until boundary is hit



Flood Fill

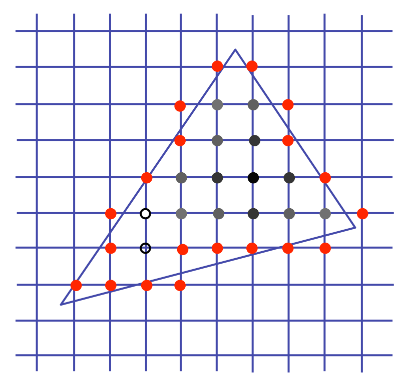
- draw edges
- run:

FloodFill(Polygon P, int x, int y, Color C)
if not (**OnBoundary**(x,y,P) or **Colored**(x,y,C))
begin
    PlotPixel(x,y,C);
    FloodFill(P,$x+1$,y,C);
    FloodFill(P,x,$y+1$,C);
    FloodFill(P,x,$y-1$,C);
    FloodFill(P,$x-1$,y,C);
end ;

- drawbacks?

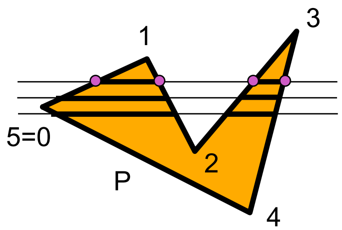
Flood Fill Drawbacks

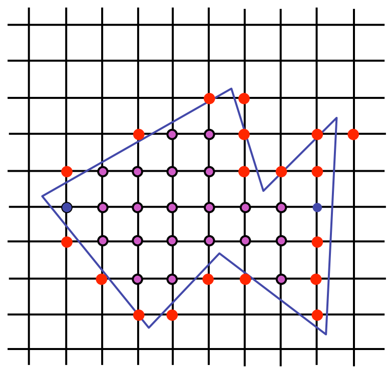
- pixels visited up to 4 times to check if already set
- need per-pixel flag indicating if set already
  - must clear for every polygon!



Scanline Algorithms

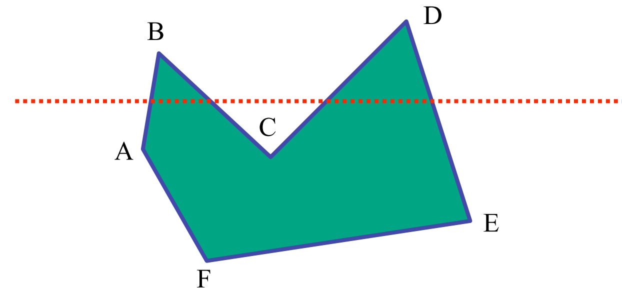
- scanline: a line of pixels in an image
  - set pixels inside polygon boundary along horizontal lines one pixel apart vertically





General Polygon Rasterization

- how do we know whether given pixel on scanline is inside or outside polygon?



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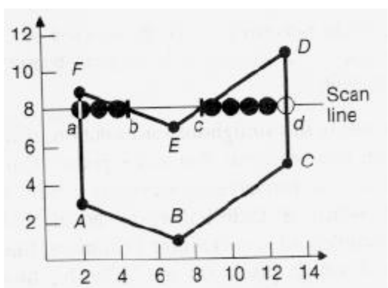
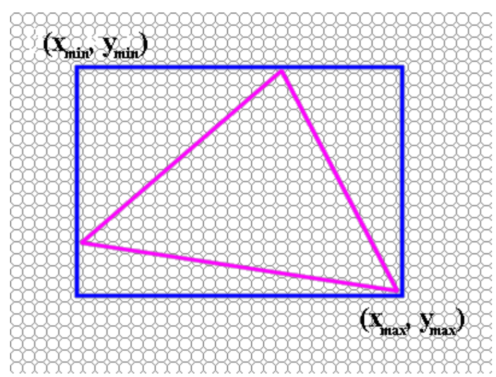


- idea: use a parity test

```
for each scanline
    edgeCnt = 0;
    for each pixel on scanline (l to r)
        if (oldpixel->newpixel crosses edge)
            edgeCnt ++;
        // draw the pixel if edgeCnt odd
        if (edgeCnt % 2)
            setPixel(pixel);
```

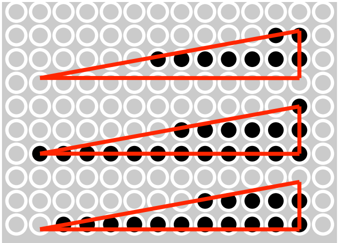

Making It Fast: Bounding Box

- smaller set of candidate pixels
  - loop over xmin, xmax and ymin,ymax instead of all x, all y

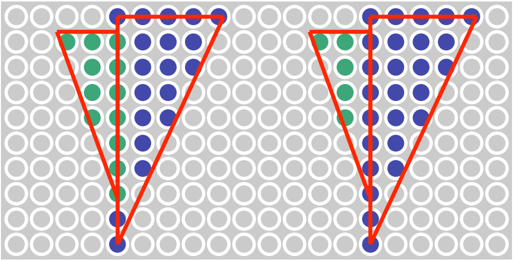


Triangle Rasterization Issues

- moving slivers



- shared edge ordering



Triangle Rasterization Issues

- *exactly which pixels should be lit?*
  - pixels with centers inside triangle edges
- *what about pixels exactly on edge?*
  - draw them: order of triangles matters (it shouldn't)
  - don't draw them: gaps possible between triangles
- need a consistent (if arbitrary) rule
  - example: draw pixels on left or top edge, but not on right or bottom edge
  - example: check if triangle on same side of edge as offscreen point

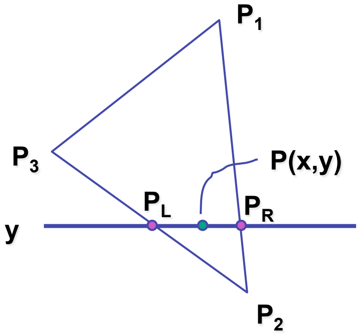
Interpolation

Interpolation During Scan Conversion

- drawing pixels in polygon requires interpolating many values between vertices
  - r,g,b colour components
    - use for shading
  - z values
  - u,v texture coordinates
  - N_x, N_y, N_z surface normals
- equivalent methods (for triangles)
  - bilinear interpolation
  - barycentric coordinates

Bilinear Interpolation

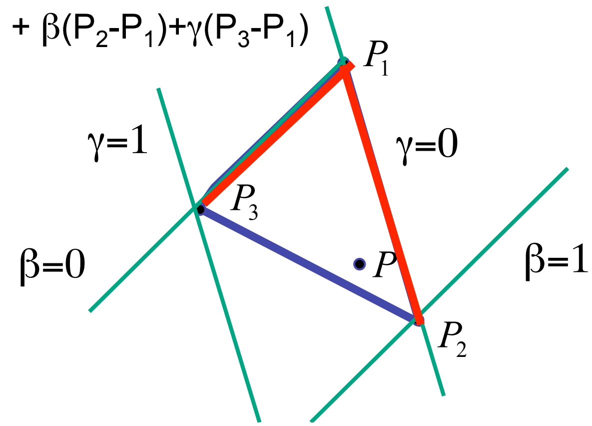
- interpolate quantity along L and R edges, as a function of y
  - then interpolate quantity as a function of x



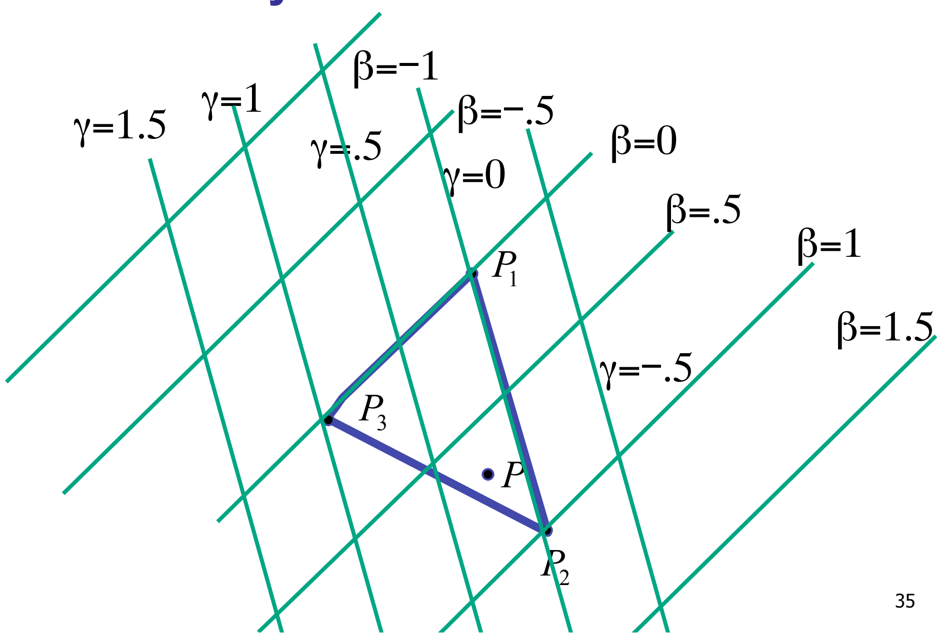
Barycentric Coordinates

- non-orthogonal coordinate system based on triangle itself
  - origin: P_1, basis vectors: (P_2-P_1) and (P_3-P_1)

$P = P_1 + \beta(P_2-P_1)+\gamma(P_3-P_1)$



Barycentric Coordinates



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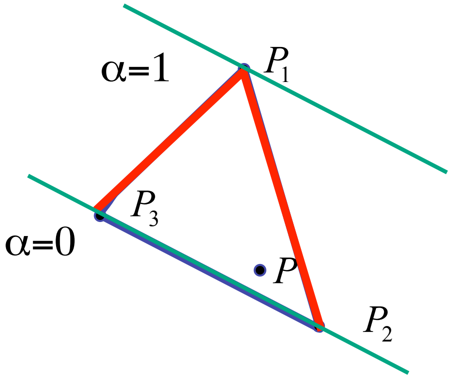
$P = P_1 + \beta(P_2-P_1)+\gamma(P_3-P_1)$

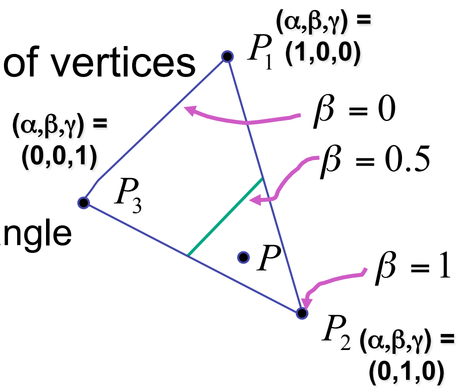
$P = (1-\beta-\gamma)P_1 + \beta P_2+\gamma P_3$

$P = \alpha P_1 + \beta P_2+\gamma P_3$



Using Barycentric Coordinates

- weighted combination of vertices
  - smooth mixing
  - speedup
    - compute once per triangle

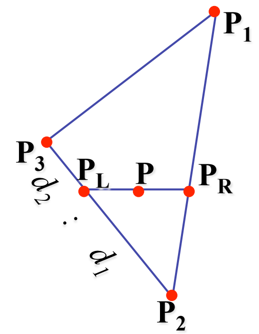


$$\left\{\begin{array}{l} P = \alpha \cdot P_1 + \beta \cdot P_2 + \gamma \cdot P_3 \\ \alpha + \beta + \gamma = 1 \\ 0 \le \alpha, \beta, \gamma \le 1 \text{ for points inside triangle} \end{array}\right.$$

"convex combination of points"

Deriving Barycentric From Bilinear

- from bilinear interpolation of point P on scanline



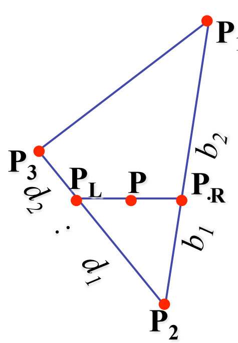
$$P_L = P_2 + \frac{d_1}{d_1 + d_2}(P_3 - P_2)$$

$$= (1 - \frac{d_1}{d_1 + d_2})P_2 + \frac{d_1}{d_1 + d_2}P_3 =$$

$$= \frac{d_2}{d_1 + d_2}P_2 + \frac{d_1}{d_1 + d_2}P_3$$

Deriving Barycentric From Bilineaer

- similarly



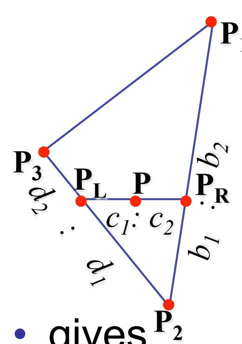
$$P_R = P_2 + \frac{b_1}{b_1 + b_2}(P_1 - P_2)$$

$$= (1 - \frac{b_1}{b_1 + b_2})P_2 + \frac{b_1}{b_1 + b_2}P_1 =$$

$$= \frac{b_2}{b_1 + b_2}P_2 + \frac{b_1}{b_1 + b_2}P_1$$

Deriving Barycentric From Bilinear

- combining



$$P = \frac{c_2}{c_1 + c_2} \cdot P_L + \frac{c_1}{c_1 + c_2} \cdot P_R$$

$$P_L = \frac{d_2}{d_1 + d_2}P_2 + \frac{d_1}{d_1 + d_2}P_3$$

$$P_R = \frac{b_2}{b_1 + b_2}P_2 + \frac{b_1}{b_1 + b_2}P_1$$

- gives

$$P = \frac{c_2}{c_1 + c_2}\left(\frac{d_2}{d_1 + d_2}P_2 + \frac{d_1}{d_1 + d_2}P_3\right) + \frac{c_1}{c_1 + c_2}\left(\frac{b_2}{b_1 + b_2}P_2 + \frac{b_1}{b_1 + b_2}P_1\right)$$

Deriving Barycentric From Bilinear

- thus $P = \alpha P_1 + \beta P_2 + \gamma P_3$ with

$$\alpha = \frac{c_1}{c_1 + c_2}\frac{b_1}{b_1 + b_2}$$

$$\beta = \frac{c_2}{c_1 + c_2}\frac{d_2}{d_1 + d_2} + \frac{c_1}{c_1 + c_2}\frac{b_2}{b_1 + b_2}$$

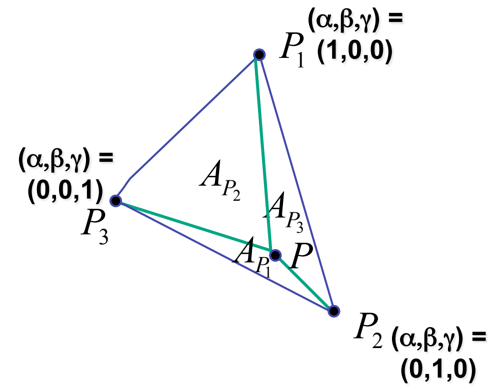
$$\gamma = \frac{c_2}{c_1 + c_2}\frac{d_1}{d_1 + d_2}$$

- can verify barycentric properties

$$\alpha + \beta + \gamma = 1, \qquad 0 \le \alpha, \beta, \gamma \le 1$$

Computing Barycentric Coordinates

- 2D triangle area
  - half of parallelogram area
    - from cross product



$A = A_{P1} + A_{P2} + A_{P3}$

$\alpha = A_{P1} / A$

$\beta = A_{P2} / A$

$\gamma = A_{P3} / A$