Shadow Volumes
Ray-Tracing

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Course News

Assignment 3 (project)
- Due April 1

Reading
- Chapter 10 (ray tracing), except 10.8-10.10
- Chapter 14 (global illumination)

Homework 8
- Out today
- Last homework…
The Rendering Pipeline

Geometry Database → Model/View Transform. → Lighting → Perspective Transform. → Clipping

Scan Conversion → Texturing → Depth Test → Blending → Frame-buffer

Rasterization → Fragment Processing

Visualizing the Shadow Mapping Technique (1)

A scene with fairly complex shadows

the point light source
Visualizing the Shadow Mapping Technique (2)

**Compare with and without shadows**

![Image with shadows](image1.png)

![Image without shadows](image2.png)

**Visualizing the Shadow Mapping Technique (3)**

**The scene from the light’s point-of-view**

![Image from light's point-of-view](image3.png)

*FYI: from the eye’s point-of-view again*
Visualizing the Shadow Mapping Technique (4)

The depth buffer from the light's point-of-view

FYI: from the light's point-of-view again

Visualizing the Shadow Mapping Technique (5)

Projecting the depth map onto the eye's view

FYI: depth map for light's point-of-view again
Visualizing the Shadow Mapping Technique (6)

*Projecting light’s planar distance onto eye’s view*

Visualizing the Shadow Mapping Technique (6)

*Comparing light distance to light depth map*

Green is where the light planar distance and the light depth map are approximately equal.

Non-green is where shadows should be.

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Visualizing the Shadow Mapping Technique (7)

Complete scene with shadows

Notice how specular highlights never appear in shadows

Notice how curved surfaces cast shadows on each other

Shadow Maps

Approach for shadows from point light sources

- Surface point is in shadow if it is not visible from the light source
- Use depth buffer to test visibility:
  - Render scene from the point light source
  - Store resulting depth buffer as texture map
  - For every fragment generated while rendering from the camera position, project the fragment into the depth texture taken from the camera, and check if it passes the depth test.
Shadow Volumes

Use new buffer: stencil buffer
- Just another channel of the framebuffer
- Can count how often a pixel is drawn

Algorithm (1):
- Generate silhouette polygons for all objects
  - Polygons starting at silhouette edges of object
  - Extending away from light source towards infinity
  - These can be computed in vertex programs

Image by ATI
Shadow Volumes

**Algorithm (2):**

- Render all original geometry into the depth buffer
  - *I.e. do not draw any colors (or only draw ambient illumination term)*

- Render front-facing silhouette polygons while incrementing the stencil buffer for every rendered fragment

- Render back-facing silhouette polygons while decrementing the stencil buffer for every rendered fragment

- Draw illuminated geometry where the stencil buffer is 0, shadow elsewhere
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**Discussion:**
- Object space method therefore no precision issues
- Lots of large polygons: can be slow
  - *High geometry count*
  - *Large number of pixels rendered*

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Ray Tracing

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Course Topics for the Rest of the Term

Ray-tracing & Global Illumination
- Today, next week

Parametric Curves/Surfaces

Overview of current research

Overview

So far
- Real-time/HW rendering w/ Rendering Pipeline
- Rendering algorithms using the Rendering Pipeline

Now
- Ray-Tracing
  - Simple algorithm for software rendering
    - Usually offline (e.g. movies etc.)
    - But: research on making this method real-time
  - Extremely flexible (new effects can easily be incorporated)

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Ray-Tracing

**Basic Algorithm (Whithead):**

for every pixel \( p_i \) {
    Generate ray \( r \) from camera position through pixel \( p_i \)
    \( p_i = \) background color
    for every object \( o \) in scene {
        if( \( r \) intersects \( o \) && intersection is closer than previously found intersections)
            Compute lighting at intersection point, using local normal and material properties; store result in \( p_i \)
    }
}

Ray-Tracing

**Issues:**

- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- Lighting and shading
- Efficient data structures so we don’t have to test intersection with every object
### Ray-Tracing – Generation of Rays

#### Camera Coordinate System
- Origin: C (camera position)
- Viewing direction: v
- Up vector: u
- x direction: \( x = v \times u \)

**Note:**
- Corresponds to viewing transformation in rendering pipeline!
- See gluLookAt...

### Ray-Tracing – Generation of Rays

#### Other parameters:
- Distance of Camera from image plane: \( d \)
- Image resolution (in pixels): \( w, h \)
- Left, right, top, bottom boundaries in image plane: \( l, r, t, b \)

**Then:**
- Lower left corner of image: \( O = C + d \cdot v + l \cdot x + b \cdot u \)
- Pixel at position \( i, j \) \( (i=0..w-1, j=0..h-1) \):
  \[
P_{i,j} = O + i \cdot \frac{r-l}{w-1} \cdot x - j \cdot \frac{t-b}{h-1} \cdot u
  = O + i \cdot \Delta x \cdot x - j \cdot \Delta y \cdot y
  \]
Ray-Tracing — Generation of Rays

Ray in 3D Space:

\[ R_{i,j}(t) = C + t \cdot (P_{i,j} - C) = C + t \cdot \mathbf{v}_{i,j} \]

where \( t = 0 \ldots \infty \)

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Ray-Tracing

Issues:

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- \textbf{Intersection of rays with geometric primitives}
- Geometric transformations
- Lighting and shading
- Efficient data structures so we don’t have to test intersection with every object
Ray Intersections

Task:
- Given an object o, find ray parameter t, such that $R_{i,j}(t)$ is a point on the object
  - Such a value for t may not exist
- Intersection test depends on geometric primitive

Spheres at origin:
- Implicit function:
  \[ S(x, y, z) : x^2 + y^2 + z^2 = r^2 \]
- Ray equation:
  \[ R_{i,j}(t) = C + t \cdot v_{i,j} = \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix} + t \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} c_x + t \cdot v_x \\ c_y + t \cdot v_y \\ c_z + t \cdot v_z \end{pmatrix} \]
Ray Intersections

To determine intersection:
- Insert ray $R_{ij}(t)$ into $S(x,y,z)$:
  \[(c_x + t \cdot v_x)^2 + (c_y + t \cdot v_y)^2 + (c_z + t \cdot v_z)^2 = r^2\]
- Solve for $t$ (find roots)
  - Simple quadratic equation

Other Primitives:
- Implicit functions:
  - Spheres at arbitrary positions
    - Same thing
  - Conic sections (hyperboloids, ellipsoids, paraboloids, cones, cylinders)
    - Same thing (all are quadratic functions!)
  - Higher order functions (e.g. tori and other quartic functions)
    - In principle the same
    - But root-finding difficult
    - Net to resolve to numerical methods
Ray Intersections

Other Primitives (cont)
- Polygons:
  - First intersect ray with plane
    - linear implicit function
  - Then test whether point is inside or outside of polygon (2D test)
  - For convex polygons
    - Suffices to test whether point in on the right side of every boundary edge
    - Similar to computation of outcodes in line clipping

Ray-Tracing

Issues:
- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
  - Lighting and shading
  - Efficient data structures so we don’t have to test intersection with every object
Ray-Tracing – Geometric Transformations

**Geometric Transformations:**

- Similar goal as in rendering pipeline:
  - *Modeling scenes convenient using different coordinate systems for individual objects*

- Problem:
  - *Not all object representations are easy to transform*
    - This problem is fixed in rendering pipeline by restriction to polygons (affine invariance!)

Ray-Tracing – Geometric Transformations

**Geometric Transformations:**

- Similar goal as in rendering pipeline:
  - *Modeling scenes convenient using different coordinate systems for individual objects*

- Problem:
  - *Not all object representations are easy to transform*
    - This problem is fixed in rendering pipeline by restriction to polygons (affine invariance!)
  - *Ray-Tracing has different solution:*
    - The ray itself is always affine invariant!
    - Thus: transform ray into object coordinates!
Ray-Tracing – Geometric Transformations

Ray Transformation:
- For intersection test, it is only important that ray is in same coordinate system as object representation
- Transform all rays into object coordinates
  - Transform camera point and ray direction by inverse of model/view matrix
Shading has to be done in world coordinates (where light sources are given)
  - Transform object space intersection point to world coordinates
  - Thus have to keep both world and object-space ray

Ray-Tracing

Issues:
- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations

- Lighting and shading
  Efficient data structures so we don’t have to test intersection with every object
Ray-Tracing Lighting and Shading

Local Effects:
- Local Lighting
  - Any reflection model possible
  - Have to talk about light sources, normals…
- Texture mapping
  - Color textures
  - Bump maps
  - Environment maps
  - Shadow maps

Ray-Tracing Local Lighting

Light sources:
- For the moment: point and directional lights
- Later: are light sources
- More complex lights are possible
  - Area lights
  - Global illumination
    - Other objects in the scene reflect light
    - Everything is a light source!
    - Talk about this on Monday
Ray-Tracing
Local Lighting

Local surface information (normal...)

- For implicit surfaces \( F(x,y,z) = 0 \): normal \( n(x, y, z) \) can be easily computed at every intersection point using the gradient

\[
\mathbf{n}(x, y, z) = \begin{pmatrix} \frac{\partial F(x, y, z)}{\partial x} \\ \frac{\partial F(x, y, z)}{\partial y} \\ \frac{\partial F(x, y, z)}{\partial z} \end{pmatrix}
\]

Example: \( F(x, y, z) = x^2 + y^2 + z^2 - r^2 \)

\[
\mathbf{n}(x, y, z) = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}
\]

Needs to be normalized!

Ray-Tracing
Local Lighting

Local surface information

- Alternatively: can interpolate per-vertex information for triangles/meshes as in rendering pipeline
  - Phong shading!
  - Same as discussed for rendering pipeline
- Difference to rendering pipeline:
  - Interpolation cannot be done incrementally
  - Have to compute Barycentric coordinates for every intersection point (e.g. plane equation for triangles)
**Ray-Tracing**

**Texture Mapping**

**Approach:**
- Works in principle like in rendering pipeline
  - Given $s$, $t$ parameter values, perform texture lookup
  - Magnification, minification just as discussed
- Problem: how to get $s$, $t$
  - Implicit surfaces often don’t have parameterization
  - For special cases (spheres, other conic sections), can use parametric representation
  - Triangles/meshes: use interpolation from vertices

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**Ray-Tracing**

**Lighting and Shading**

**Global Effects**
- Shadows
- Reflections/refractions
Ray-Tracing
Shadows

**Approach:**
- To test whether point is in shadow, send out *shadow rays* to all light sources
  - *If ray hits another object, the point lies in shadow*

Ray-Tracing
Reflections/Refractions

**Approach:**
- Send rays out in reflected and refracted direction to gather incoming light
- That light is multiplied by local surface color and Fresnel term, and added to result of local shading
Recursive Ray Tracing

**Ray tracing can handle**
- Reflection (chrome)
- Refraction (glass)
- Shadows

**Spawn secondary rays**
- Reflection, refraction
  - If another object is hit, recurse to find its color
- Shadow
  - Cast ray from intersection point to light source, check if intersects another object
**Recursive Ray-Tracing Algorithm**

```plaintext
RayTrace(r,scene)
obj := FirstIntersection(r,scene)
if (no obj) return BackgroundColor;
else begin
  if (Reflect(obj)) then
    reflect_color := RayTrace(ReflectRay(r,obj));
  else
    reflect_color := Black;
  if (Transparent(obj)) then
    refract_color := RayTrace(RefractRay(r,obj));
  else
    refract_color := Black;
  return Shade(reflect_color,refract_color,obj);
end;
```

**Algorithm Termination Criteria**

**Termination criteria**
- No intersection
- Reach maximal depth
  - Number of bounces
- Contribution of secondary ray attenuated below threshold
  - Each reflection/refraction attenuates ray
Reflection

**Mirror effects**
- Perfect specular reflection

![Mirror effects image](image)

Refraction

**Happens at interface between transparent object and surrounding medium**
- E.g. glass/air boundary

**Snell's Law**
- $c_1 \sin \theta_1 = c_2 \sin \theta_2$
- Light ray bends based on refractive indices $c_1$, $c_2$

![Refraction image](image)
Total Internal Reflection

As the angle of incidence increases from 0 to greater angles ...

...the refracted ray becomes dimmer (there is less refraction)
...the reflected ray becomes brighter (there is more reflection)
...the angle of refraction approaches 90 degrees until finally
a refracted ray can no longer be seen.

Ray-Tracing Example Images

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Ray-Tracing Terminology

**Terminology:**
- Primary ray: ray starting at camera
- Shadow ray
- Reflected/refracted ray
- Ray tree: all rays directly or indirectly spawned off by a single primary ray

**Note:**
- Need to limit maximum depth of ray tree to ensure termination of ray-tracing process!

Ray-Tracing

**Issues:**
- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- Lighting and shading
- **Efficient data structures so we don’t have to test intersection with every object**
Ray Tracing

**Data Structures**
- Goal: reduce number of intersection tests per ray
- Lots of different approaches:
  - (Hierarchical) bounding volumes
  - Hierarchical space subdivision
    - Oct-tree, k-D tree, BSP tree

Bounding Volumes

**Idea:**
- Rather than testing every ray against a potentially very complex object (e.g., triangle mesh), do a quick conservative test first which eliminates most rays
  - Surround complex object by simple, easy to test geometry (typically sphere or axis-aligned box)
    - Want to make bounding volume as tight as possible!
Hierarchical Bounding Volumes

*Extension of previous idea:*
- Use bounding volumes for groups of objects

Spatial Subdivision Data Structures

*Bounding Volumes:*
- Find simple object completely enclosing complicated objects
  - *Boxes, spheres*
- Hierarchically combine into larger bounding volumes

*Spatial subdivision data structure:*
- Partition the whole space into cells
  - *Grids, oct-trees, (BSP trees)*
- Simplifies and accelerates traversal
- Performance less dependent on order in which objects are inserted
Regular Grid

**Subdivide space into rectangular grid:**
- Associate every object with the cell(s) that it overlaps with
- Find intersection: traverse grid

In 3D: regular grid of cubes (voxels):

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Creating a Regular Grid

**Steps:**
- Find bounding box of scene
- Choose grid resolution in x, y, z
- Insert objects
- Objects that overlap multiple cells get referenced by all cells they overlap
Grid Traversal

**Traversal:**
- Start at ray origin
- While no intersection found
  - Go to next grid cell along ray
  - Compute intersection of ray with all objects in the cell
  - Determine closest such intersection
  - Check if that intersection is inside the cell
  - If so, terminate search

**Traversals**

**Note:**
- This algorithm calls for computing the intersection points multiple times (once per grid cell)
- In practice: store intersections for a (ray, object) pair once computed, reuse for future cells

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Regular Grid Discussion

**Advantages?**
- Easy to construct
- Easy to traverse

**Disadvantages?**
- May be only sparsely filled
- Geometry may still be clumped

Adaptive Grids

- Subdivide until each cell contains no more than \( n \) elements, or maximum depth \( d \) is reached

Nested Grids

Octree/(Quadtree)

This slide and the next are curtesy of Fredo Durand at MIT

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Primitives in an Adaptive Grid

- Can live at intermediate levels, or be pushed to lowest level of grid

Octree/(Quadtree)

Adaptive Grid Discussion

**Advantages**
- Grid complexity matches geometric density

**Disadvantages**
- More expensive to traverse than regular grid
Coming Up...

*Wednesday:*
- More ray-tracing

*Next Week:*
- Global illumination