**Sampling & Reconstruction**
Wolfgang Heidrich

---

**Course News**

*Assignment 3*
- Due April 1

*Homework 6*
- Out today
- Questions on texture mapping

*Reading*
- Chapter 11 (Texture Mapping)

*Quiz 2*
- On Wednesday, Mar 9
- Topics: full Rendering Pipeline, except transformations

---

**The Rendering Pipeline**

**Texture Lookup – Sampling & Reconstruction**

- How to deal with:
  - Pixels that are much larger than texels?
    - Apply filtering, "averaging"
    - "Minification"
  - Pixels that are much smaller than texels?
    - Interpolate
    - "Magnification"

---

**Magnification: Interpolating Textures**

- Nearest neighbor
- Bilinear
- Hermite (cubic)

---

**Minification: MIPmapping**

- Use "image pyramid" to precompute averaged versions of the texture
- Store whole pyramid in single block of memory
MIPmaps

Multum in parvo
“many things in a small place”
Series of prefiltered texture maps of decreasing resolutions
Avoid shimmering and flashing as objects move

`gluBuild2DMipmaps`
Automatically constructs a family of textures from original texture size down to 1x1

MIPmap storage

Only 1/3 more space required

Sampling & Reconstruction

CPSC 314

Samples

- Most things in the real world are continuous
- Everything in a computer is discrete
- The process of mapping a continuous function to a discrete one is called sampling
- The process of mapping a discrete function to a continuous one is called reconstruction
- The process of mapping a continuous variable to a discrete one is called quantization
- Rendering an image requires both sampling and quantization
- Displaying an image involves reconstruction

Line Segments

- We tried to sample a line segment so it would map to a 2D raster display
- We quantized the pixel values to 0 or 1
- We saw stair steps, or jaggies

Line Segments

- Instead, quantize to many shades
- But what sampling algorithm is used?
**Unweighted Area Sampling**

Shade pixels wrt area covered by thickened line
Equal areas cause equal intensity, regardless of distance from pixel center to area
- Rough approximation formulated by dividing each pixel into a finer grid of pixels

Primitive cannot affect intensity of pixel if it does not intersect the pixel

**Weighted Area Sampling**

Intuitively, pixel cut through the center should be more heavily weighted than one cut along corner

Weighting function, \( W(x,y) \)

Specifies the contribution of primitive passing through the point \( (x, y) \) from pixel center

![Intensity graph](Intensity.png)

**Images**

An image is a 2D function \( I(x, y) \)

- Specifies intensity for each point \( (x, y) \)
- We consider each color channel independently

![Image sample](ImageSample.png)

**Image Sampling and Reconstruction**

- Convert continuous image to discrete set of samples
- Display hardware reconstructs samples into continuous image
  - Finite sized source of light for each pixel

![Sampling diagram](SamplingDiagram.png)

**Point Sampling an Image**

- Simplest sampling is on a grid
- Sample depends solely on value at grid points

![Sampling grid](SamplingGrid.png)

**Point Sampling**

Multiply sample grid by image intensity to obtain a discrete set of points, or samples.
Sampling Errors

Some objects missed entirely, others poorly sampled
Could try unweighted or weighted area sampling
But how can we be sure we show everything?

Need to think about entire class of solutions!

Image As Signal

Image as spatial signal
2D raster image
- Discrete sampling of 2D spatial signal
1D slice of raster image
- Discrete sampling of 1D spatial signal

Pixel position across scanline
Examples from Foley, van Dam, Feiner, and Hughes

Sampling Theory

How would we generate a signal like this out of simple building blocks?

Theorem
- Any signal can be represented as an (infinite) sum of sine waves at different frequencies

Sampling Theory in a Nutshell

Terminology
- Wavelength – length of repeated sequence on infinite signal
- Frequency – 1/wavelength (number of repeated sequences in unit length)

Example – sine wave
- Wavelength = 2π
- Frequency = 1/2π

Fourier Transform – Summing Sinusoids

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Waveform</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>sin(x)</td>
</tr>
<tr>
<td>3</td>
<td>sin(3x) / 3</td>
</tr>
<tr>
<td>5</td>
<td>sin(5x) / 5</td>
</tr>
<tr>
<td>7</td>
<td>sin(7x) / 7</td>
</tr>
<tr>
<td>9</td>
<td>sin(9x) / 9</td>
</tr>
</tbody>
</table>

Fourier Transform
1D Sampling and Reconstruction

Problems
- Jaggies – abrupt changes
- Lose data
**Sampling Theorem**

- Continuous signal can be completely recovered from its samples

- Sampling rate greater than twice highest frequency present in signal

  - **Claude Shannon**

---

**Nyquist Rate**

- **Lower bound on sampling rate**
  - Twice the highest frequency component in the image's spectrum

---

**Falling Below Nyquist Rate**

- *When sampling below Nyquist Rate, resulting signal looks like a lower-frequency one*

  - This is **Aliasing**

  ![Aliasing](image)

  *Fig. 14.17 Sampling below the Nyquist rate. (Courtesy of George Wolberg, Caltech)*

---

**Aliasing**

- Incorrect appearance of high frequencies as low frequencies

  - **To avoid:** anti-aliasing

    - Supersample
      - Sample at higher frequency
    - Low pass filtering
      - Remove high frequency function parts
      - Aka prefiltering, band-limiting

---

**Supersampling**
Low-Pass Filtering

Previous Antialiasing Example

Texture mipmapping: low pass filter

(a)    (b)

Discussion

Sampling & Reconstruction
- Fundamental issue in graphics, vision, and many other areas of computer science
- Whenever continuous signals need to be represented in a computer
- Aliasing refers to the problem of reconstruction errors due to frequencies above the Nyquist limit
- These frequencies show up as erroneous low frequency content

Anti-Aliasing Approaches
- Low-pass filtering (before sampling!)
  - Avoids aliasing
  - May not be practical in all settings
  - For images: artifacts around edges?!
- Supersampling
  - General algorithmic approach
  - However: even the higher resolution image has a Nyquist limit!
  - Slow

Discussion

Coming Up:

Friday
- Sampling & reconstruction