Transformations of Normal Vectors
Intro to Lighting

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Course News

Assignment 1
Due Monday!

Homework 2
Discussed in labs this week

The Rendering Pipeline

Geometry Database

Model/View Transform

Lighting

Perspective Transform

Clipping

Scan Conversion

Texturing

Depth Test

Blending

Frame-buffer

Normals & Affine Transformations

Question:
If we transform some geometry with an affine transformation, how does that affect the normal vector?

Consider:
Rotation
Translation
Scaling
Shear

Homogeneous Planes And Normals

Planes In Cartesian Coordinates:

\{(x,y,z) \mid n_x x + n_y y + n_z z + d = 0\}

- \(n_x, n_y, n_z\) and \(d\) are the parameters of the plane (normal and distance from origin)
- \(d\) is positive
- \(n\) point to half-space containing origin

Planes In Homogeneous Coordinates:

\{(x,y,z,1) \mid n_x x + n_y y + n_z z + d \cdot 1 = 0\}

Want:
Representation for normals that allows us to easily describe how they change under affine transformation

Why?
Normal vectors will be of special interest when we talk about lighting (next week)
Homogeneous Planes And Normals

**Planes in homogeneous coordinates are represented as row vectors**

$$E = [n_x, n_y, n_z, d]$$

Condition that a point $$[x, y, z]^T$$ is located in $$E$$

$$
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} \in E = [n_x, n_y, n_z, d] \Leftrightarrow [n_x, n_y, n_z, d] \cdot
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} = 0
$$

Homogeneous Planes And Normals

**Transformations of planes**

$$[n_x, n_y, n_z, d] \cdot
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} = 0 \Leftrightarrow T([n_x, n_y, n_z, d]) \cdot (A \cdot
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}) = 0$$

Thus, planes have to be transformed by the inverse of the affine transformation (multiplied from left as a row vector)!

Homogeneous Planes And Normals

**Homogeneous Normals**

- The plane definition also contains its normal
- Normal written as a vector $$[n_x, n_y, n_z, 0]^T$$

$$
\begin{bmatrix}
\vec{v}_1 \\
\vec{v}_2 \\
\vec{v}_3 \\
0
\end{bmatrix} = 0 \Leftrightarrow ((A^{-T} \cdot
\begin{bmatrix}
\vec{v}_1 \\
\vec{v}_2 \\
\vec{v}_3 \\
0
\end{bmatrix}) \cdot (A \cdot
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix})) = 0
$$

Thus, the normal to any surface has to be transformed by the inverse transpose of the affine transformation (multiplied from the right as a column vector)!

Transforming Homogeneous Normals

**Inverse Transpose of**

- Rotation by $$\alpha$$
  - Rotation by $$\alpha$$
- Scale by $$s$$
  - Scale by $$1/s$$
- Translation by $$t$$
  - Identity matrix!
- Shear by a along $$x$$ axis
  - Shear by $$-a$$ along $$y$$ axis

Intro to Lighting

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**Lighting**

**Goal**
- Model the interaction of light with surfaces to render realistic images

**Contributing Factors**
- Light sources
  - Shape and color
- Surface materials
  - How surfaces reflect light

**Materials**

**Analyzing surface reflectance**
- Illuminate surface point with a ray of light from different directions
- Observe how much light is reflected in all possible directions

**Does this tell us anything about general lighting conditions?**

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**Materials**

**Light is linear**
- If two rays illuminate the surface point the result is just the sum of the individual reflections for each ray
- For N directions the reflection is the sum of the individual N reflections
- For light arriving from a continuum of directions, the reflection is the integral over the reflections caused by the individual directions
  - More on this when we talk about global illumination at the end of the term

**Experiment**

**Goal:**
- Visualize reflected light distribution for a given illuminating ray

**Physical setup:**
- Laser illumination
- Water tank with fluorescent dye
  - Makes laser light visible as it travels through “empty” space

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**Material Examples**
Diffuse Material

Glossy Material

Specular Material

Glossy + Specular

Glossy + Specular

**BRDF**

*Model for all these effects:*

- Bi-directional
  - i.e. dependent on 2 directions: incident, exitant
- Reflectance
  - A model for surface reflection (not transmission)
- Distribution
  - Light is distributed over different exitant directions
- Function
BRDF measurement

Limitations of the BRDF Model

BRDFs cannot describe
- Light of one wavelength that gets absorbed and re-emitted at a different wavelength
  (fluorescence)
- Light that gets absorbed and emitted much later
  (phosphorescence)
- Light that penetrates the object surface, scatters in the interior of the object, and exits at a different point form
  where it entered
  (subsurface scattering)

Materials

Practical Considerations
- In practice, we often simplify the BRDF model further
- Derive specific formulas that describe different reflectance behaviors
  - E.g. diffuse, glossy, specular
- Computational efficiency is also a concern

Coming Up:

Next week
- More on lighting / shading