This final problem sheet deals with ray-tracing and stochastic sampling. The solutions will be published online, and discussed in the labs on March 28/29 (labs in the second half of that week will be used for project demos).

1 Ray-Tracing

Sketch the ray-tree for the scene and primary ray depicted below up to level 3 (the primary ray is level 0). Assume the following material properties:

Surface A : transparent and specular surface (like glass). No diffuse or Phong contribution.
Surface B : Diffuse and Phong reflection, but not specular or transparent.
Surface C : Diffuse, Phong, and specular reflection, no transparency.
2 Monte Carlo Sampling

Assume you are given a bi-variate function $f(x, y)$, and want to compute the integral

$$\int_{a}^{b} \int_{c}^{d} f(x, y)dx\,dy.$$ 

Unfortunately, you cannot compute this integral analytically since $f$ is too complicated. You can only evaluate $f(x, y)$ for specific values of $x$ and $y$.

How would you evaluate the above integral using Monte Carlo sampling? First describe the general idea, then give a specific formula for the above case.