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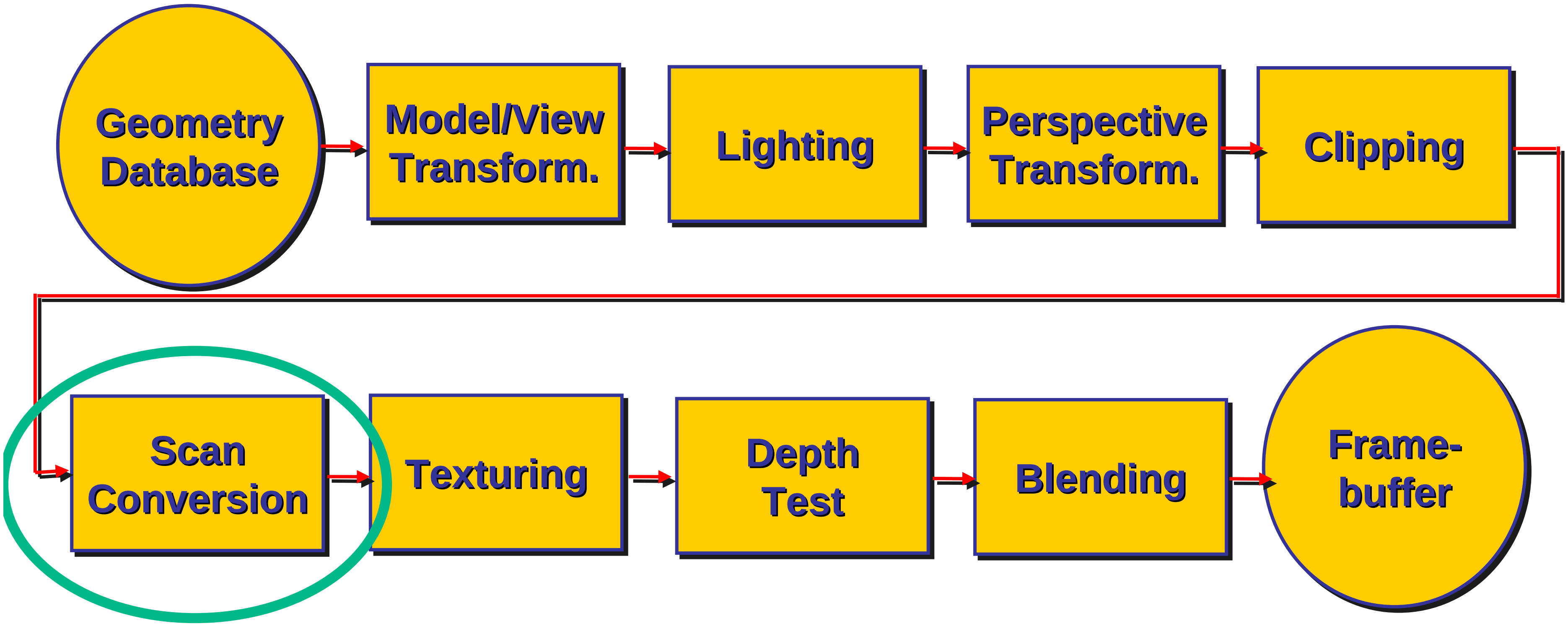
Chapter 7

Scan Conversion – Drawing on Raster Display (part 1 – Lines)

The Rendering Pipeline



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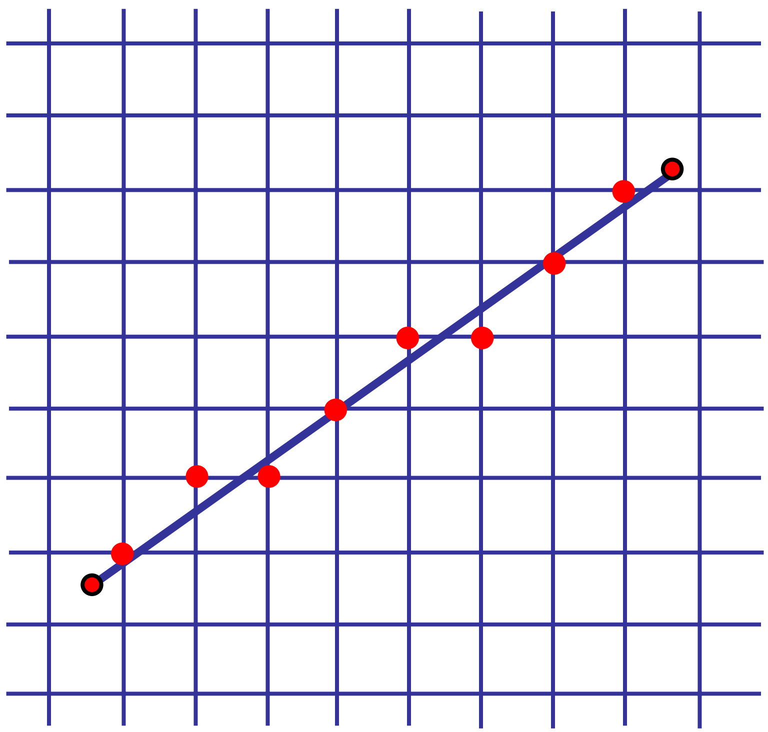
Scan Conversion - Rasterization

- Convert continuous rendering primitives into discrete fragments/pixels
  - Lines
    - Bresenham
  - Triangles
    - Flood Fill
    - Scanline
    - Implicit formulation

Scan Conversion - Lines

- Given segment equation fill in the pixels
  - In drawings below – grid points = centers of pixels



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Lines and Curves

- Explicit - one coordinate as function of the others

$$y = f(x)$$

$$z = f(x, y)$$

line

$$y = mx + b$$

$$y = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) + y_1$$

circle

$$y = \pm\sqrt{r^2 - x^2}$$

Lines and Curves

- Parametric – all coordinates as functions of common parameter

$$(x, y) = (f_1(t), f_2(t))$$

$$(x, y, z) = (f_1(u,v), f_2(u,v), f_3(u,v))$$

line

$$x(t) = x_1 + t(x_2 - x_1)$$
$$y(t) = y_1 + t(y_2 - y_1)$$
$$t \in [0,1]$$

circle

$$x(\theta) = r\cos(\theta)$$
$$y(\theta) = r\sin(\theta)$$
$$\theta \in [0, 2\pi]$$

Lines and Curves

- Implicit - define as "zero set" of function of all the parameters

$$\{(x,y):F(x,y)=0\}$$

$$\{(x,y,z):F(x,y,z)=0\}$$

- Defines partition of space

$$\{(x,y):F(x,y)>0\},\{(x,y):F(x,y)=0\},\{(x,y):F(x,y)<0\}$$

Lines and Curves - Implicits

line		**circle**	
$dy = y_2 - y_1$			
$dx = x_2 - x_1$			
$F(x,y) = (x - x_1)dy - (y - y_1)dx$		$F(x,y) = x^2 + y^2 - r^2$	
$F(x,y) = 0$	**(x,y) is on line**	$F(x,y) = 0$	**(x,y) is on circle**
$F(x,y) > 0$	**(x,y) is below line**	$F(x,y) > 0$	**(x,y) is outside**
$F(x,y) < 0$	**(x,y) is above line**	$F(x,y) < 0$	**(x,y) is inside**

$$F(x,y) = xdy - ydx + (y_1dx - x_1dy)$$

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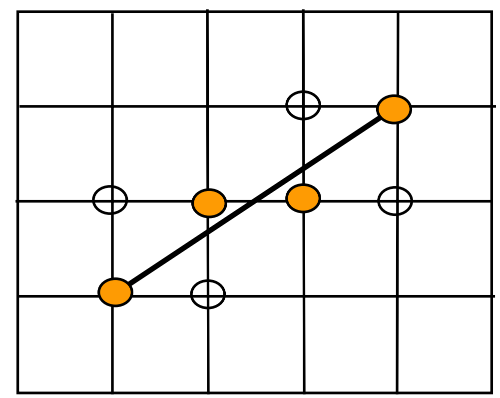
Basic Line Drawing

Assume $x_1 < x_2$ & line slope absolute value is ≤ 1

Line (x_1, y_1, x_2, y_2)
begin
float $dx, dy, x, y, slope$;
$dx \Leftarrow x_2 - x_1$;
$dy \Leftarrow y_2 - y_1$;
$slope \Leftarrow dy/dx$;
$y \Leftarrow y_1$
for x from x_1 to x_2 do
begin
  PlotPixel (x, **Round** (y)) ;
  $y \Leftarrow y + slope$;
end ;
end ;



Questions:
Can this algorithm use integer arithmetic ?

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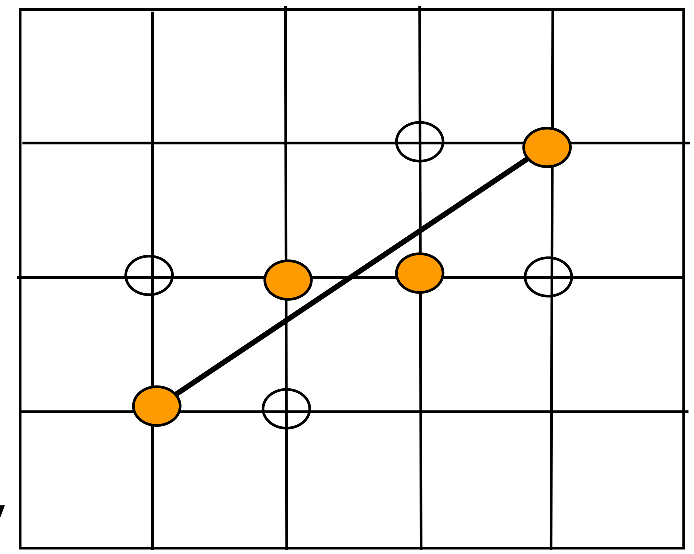
Midpoint (Bresenham) Algorithm

- **Assumptions:**

$$x_2 > x_1, y_2 > y_1 \text{ and } \frac{dy}{dx} = \frac{y_2 - y_1}{x_2 - x_1} < 1$$

- **Idea:**
  - Proceed along the line incrementally
  - Have ONLY 2 choices
  - Select one that minimizes error (distance to line)



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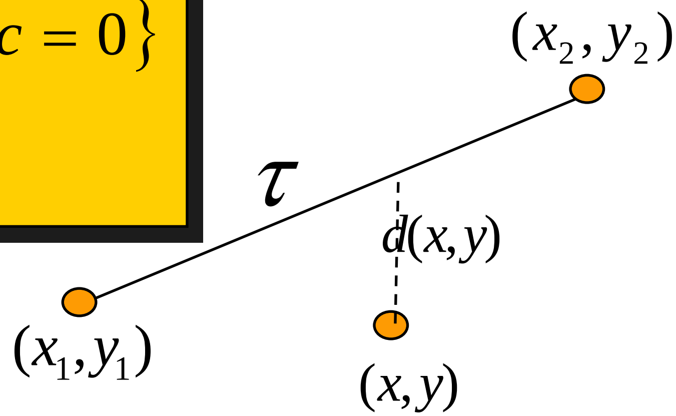
Bresenham Algorithm

Distance (error):

$$\tau = \{(x, y) : ax + by + c = xdy - ydx + c = 0\}$$

$$d(x, y) = 2(xdy - ydx + c)$$



- Given point $P = (x, y)$, $d(x, y)$ is signed distance of *p* to τ (up to normalization factor)
- *d* is zero for $P \in \tau$

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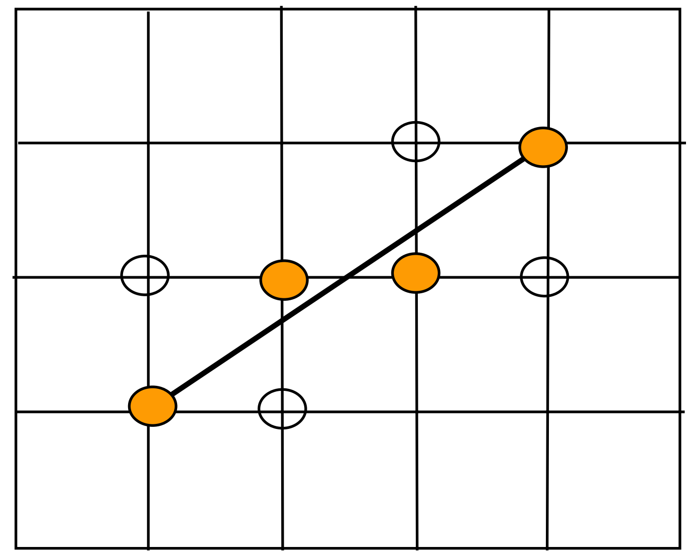
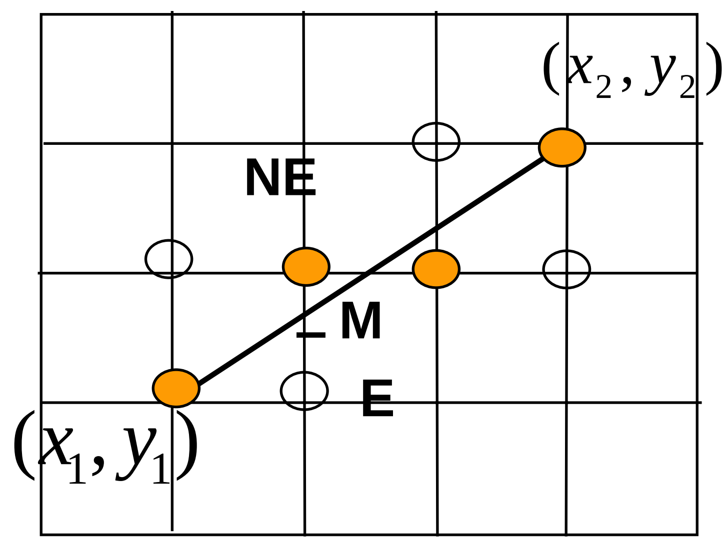
Midpoint Line Drawing (cont'd)

- Starting point satisfies $d(x_1, y_1) = 0$
- Each step moves right (east) or upper right (northeast)
- Sign of $d(x + 1; y + \frac{1}{2})$ indicates if to move east or northeast





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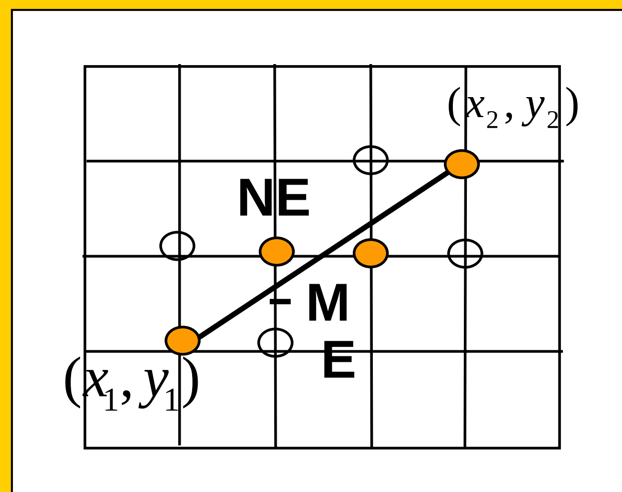
Midpoint Line Algorithm (version 1)

```
Line (x1, y1, x2, y2)
begin
int x, y, dx, dy, d;
x ⇐ x1;            y ⇐ y1;
dx ⇐ x2 − x1;      dy ⇐ y2 − y1;
PlotPixel (x, y);
while (x < x2) do
    d = (2x + 2)dy - (2y + 1)dx + 2c; // 2((x + 1)dy - (y + .5)dx + c)
    if (d < 0) then
    begin
        x ⇐ x + 1;
    end ;
    else begin
        x ⇐ x + 1;
        y ⇐ y + 1;
    end ;
    PlotPixel (x, y);
end ;
end ;
```



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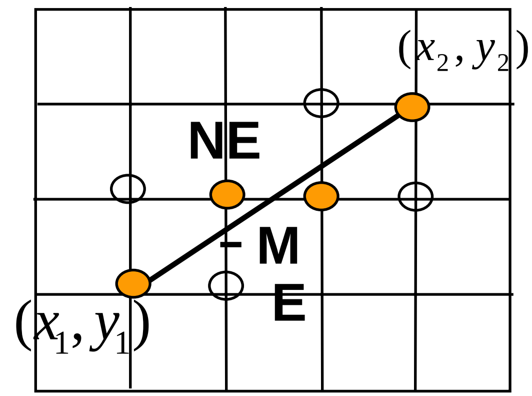
Midpoint Line Drawing (cont'd)

- Insanely efficient version (less computations inside the loop)
  – compute d incrementally



- At (x_1, y_1)

$$d_{start} = d(x_1 + 1, y_1 + \tfrac{1}{2}) = 2dy - dx$$

- Increment in *d* (after each step)
  - If move east $\Delta_e = d(x+2, y+\tfrac{1}{2}) - d(x+1, y+\tfrac{1}{2}) =$

$$2((x+2)dy - (y+\tfrac{1}{2})dx + c)) - 2((x+1)dy - (y+\tfrac{1}{2})dx + c) = 2dy$$

  - If move northeast $\Delta_{ne} = d(x_1+2, y_1+\tfrac{3}{2}) - d(x_1+1, y_1+\tfrac{1}{2}) =$

$$2((x+2)dy - (y+\tfrac{3}{2})dx + c)) - 2((x+1)dy - (y+\tfrac{1}{2})dx + c) = 2(dy - dx)$$

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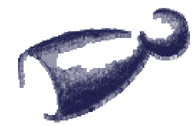
Midpoint Line Algorithm

Line (x_1, y_1, x_2, y_2)
begin
int $x, y, dx, dy, d, \Delta_e, \Delta_{ne}$;
$x \Leftarrow x_1$; $y \Leftarrow y_1$;
$dx \Leftarrow x_2 - x_1$; $dy \Leftarrow y_2 - y_1$;
$d \Leftarrow 2 * dy - dx$;
$\Delta_e \Leftarrow 2 * dy$; $\Delta_{ne} \Leftarrow 2 * (dy - dx)$;
PlotPixel (x, y) ;
while $(x < x_2)$ do
    if $(d < 0)$ then
     begin
          $d \Leftarrow d + \Delta_e$;
          $x \Leftarrow x + 1$;
    end ;
     else begin
          $d \Leftarrow d + \Delta_{ne}$;
          $x \Leftarrow x + 1$;
          $y \Leftarrow y + 1$;
    end ;
     PlotPixel (x, y) ;
end ;
end ;



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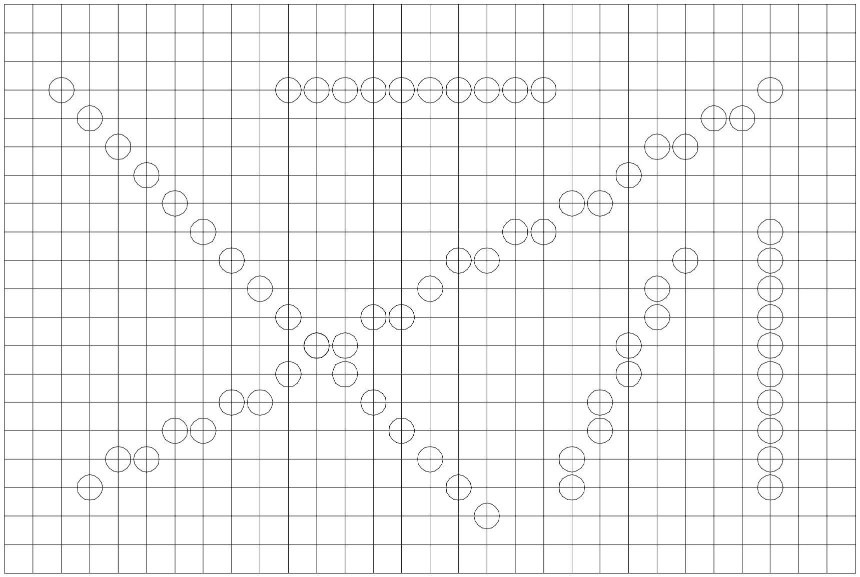
Midpoint Examples

- Question: Is there a problem with this algorithm (horizontal vs. diagonal lines)?



- Comment: extends to higher order curves – e.g. circles

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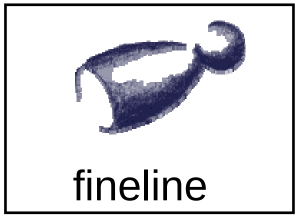
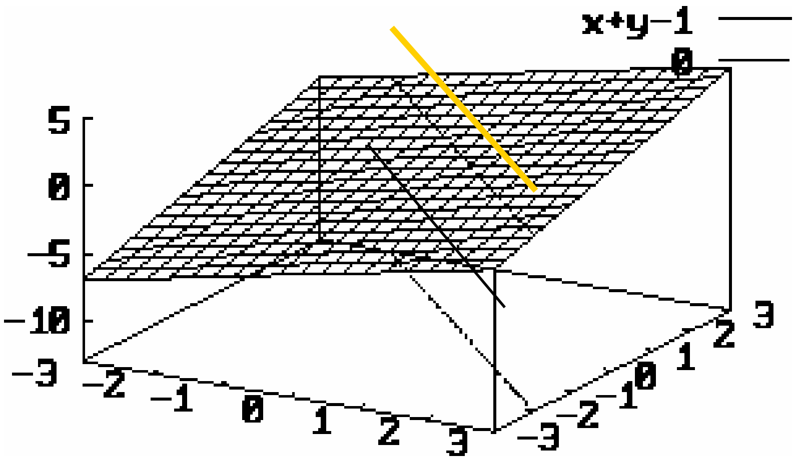


Error Function Intuition

- Error function d can be viewed as explicit surface:



$$d(x,y)=2(xdy-ydx+c)$$

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