



Tamara Munzner

## Viewing/Projections IV

Week 4, Fri Feb 1

<http://www.ugrad.cs.ubc.ca/~cs314/Vjan2008>

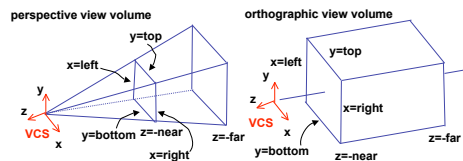
## News

- extra TA office hours in lab next week to answer questions
  - Mon 1-3
  - Tue 2-4
  - Wed 1-3
- reminder
  - Wed 2/6: Homework 1 due 1pm sharp
  - Wed 2/6: Project 1 due 6pm.

2

## Review: View Volumes

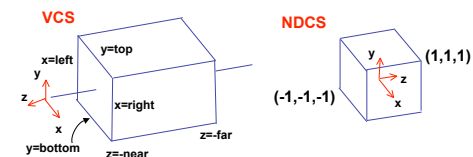
- specifies field-of-view, used for clipping
- restricts domain of  $z$  stored for visibility test



3

## Review: Understanding Z

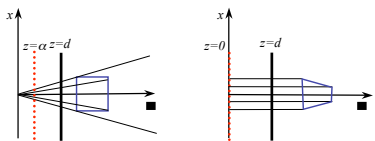
- $z$  axis flip changes coord system handedness
- RHS before projection (eye/view coords)
- LHS after projection (clip, norm device coords)



4

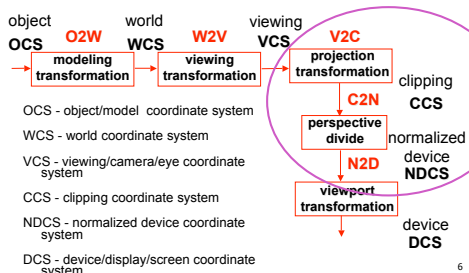
## Review: Projection Normalization

- warp perspective view volume to orthogonal view volume
  - render all scenes with orthographic projection!
  - aka perspective warp



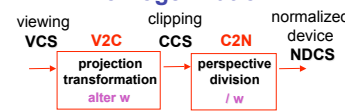
5

## Review: Projective Rendering Pipeline



6

## Review: Separate Warp From Homogenization



- warp requires only standard matrix multiply
  - distort such that orthographic projection of distorted objects is desired persp projection
    - $w$  is changed
  - clip after warp, before divide
  - division by  $w$ : homogenization

7

## Reading for Viewing

- FCG Chapter 7 Viewing
- FCG Section 6.3.1 Windowing Transforms
- RB rest of Chap Viewing
- RB rest of App Homogeneous Coords

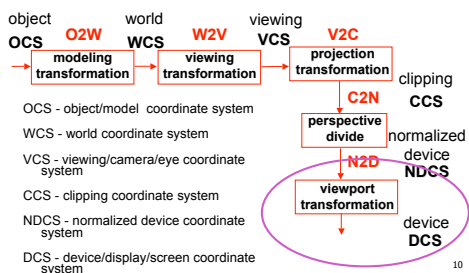
8

## Reading for Next Time

- RB Chap Color
- FCG Sections 3.2-3.3
- FCG Chap 20 Color
- FCG Chap 21.2.2 Visual Perception (Color)

9

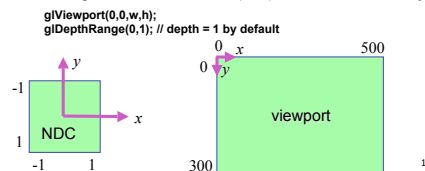
## Projective Rendering Pipeline



10

## NDC to Device Transformation

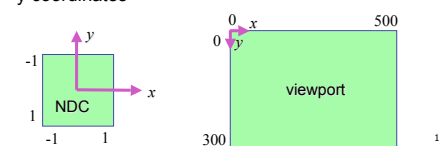
- map from NDC to pixel coordinates on display
  - NDC range is  $x = -1...1, y = -1...1, z = -1...1$
  - typical display range:  $x = 0...500, y = 0...300$ 
    - maximum is size of actual screen
    - $z$  range max and default is  $(0, 1)$ , use later for visibility



11

## Origin Location

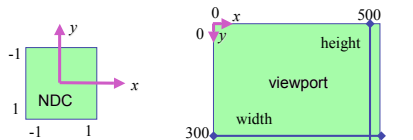
- yet more (possibly confusing) conventions
  - OpenGL origin: lower left
  - most window systems origin: upper left
- then must reflect in  $y$
- when interpreting mouse position, have to flip your  $y$  coordinates



12

## N2D Transformation

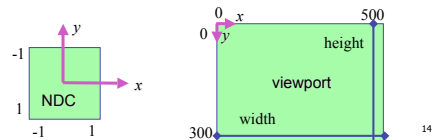
- general formulation
  - reflect in  $y$  for upper vs. lower left origin
  - scale by width, height, depth
  - translate by  $width/2, height/2, depth/2$ 
    - FCG includes additional translation for pixel centers at  $(.5, .5)$  instead of  $(0,0)$



13

## N2D Transformation

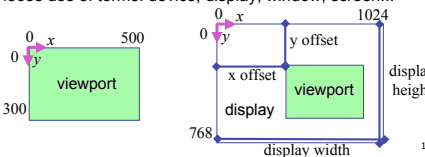
$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \frac{width}{2} - 1 \\ 0 & 1 & 0 & \frac{height}{2} - 1 \\ 0 & 0 & 1 & \frac{depth}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{width(x_w + 1) - 1}{2} \\ \frac{height(-y_w + 1) - 1}{2} \\ \frac{depth(z_w + 1)}{2} \\ 1 \end{bmatrix}$$



14

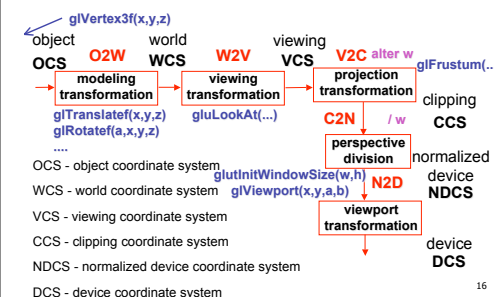
## Device vs. Screen Coordinates

- viewport/window location wrt actual display not available within OpenGL
  - usually don't care
    - use relative information when handling mouse events, not absolute coordinates
  - could get actual display height/width, window offsets from OS
- loose use of terms: device, display, window, screen...



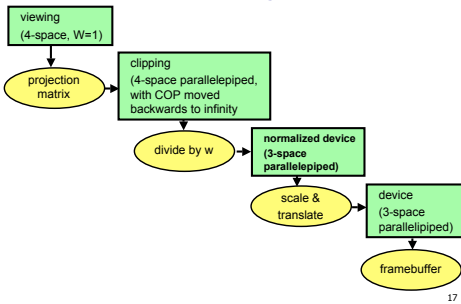
15

## Projective Rendering Pipeline



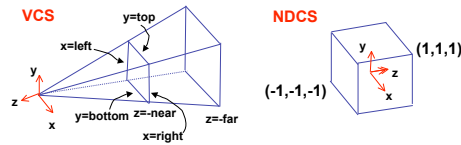
16

## Coordinate Systems



17

## Perspective To NDCS Derivation



18

## Perspective Derivation

simple example earlier:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

complete: shear, scale, projection-normalization

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ F & 1 & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

19

## Perspective Derivation

earlier:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

complete: shear, scale, projection-normalization

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ F & 1 & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

20

## Perspective Derivation

earlier:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

complete: shear, scale, projection-normalization

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ F & 1 & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

21

## Perspective Derivation

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$x' = Ex + Az$      $x = \text{left} \rightarrow x'/w' = 1$   
 $y' = Fy + Bz$      $x = \text{right} \rightarrow x'/w' = -1$   
 $z' = Cz + D$      $y = \text{top} \rightarrow y'/w' = 1$   
 $w' = -z$      $y = \text{bottom} \rightarrow y'/w' = -1$   
 $z = -\text{near} \rightarrow z'/w' = 1$   
 $z = -\text{far} \rightarrow z'/w' = -1$

$$y' = Fy + Bz, \quad \frac{y'}{w'} = \frac{Fy + Bz}{-z}, \quad 1 = \frac{Fy + Bz}{-z}, \quad 1 = \frac{Fy + Bz}{-z}$$

$$1 = F \frac{y}{-z} + B \frac{z}{-z}, \quad 1 = F \frac{y}{-z} - B, \quad 1 = F \frac{\text{top}}{(-\text{near})} - B$$

$$1 = F \frac{\text{top}}{\text{near}} - B$$

22

## Perspective Derivation

- similarly for other 5 planes
- 6 planes, 6 unknowns

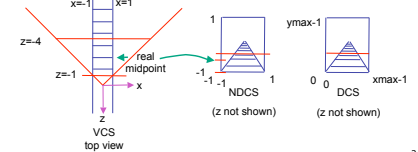
$$\begin{bmatrix} 2n & 0 & r+l & 0 \\ r-l & 2n & r-l & 0 \\ 0 & t-b & t-b & 0 \\ 0 & 0 & -(f+n) & -2fn \\ 0 & 0 & f-n & f-n \end{bmatrix}$$

23

## Perspective Example

tracks in VCS:  
left  $x=-1, y=-1$   
right  $x=1, y=-1$

view volume  
left = -1, right = 1  
bot = -1, top = 1  
near = 1, far = 4



24

## Perspective Example

view volume  
• left = -1, right = 1  
• bot = -1, top = 1  
• near = 1, far = 4

$$\begin{bmatrix} 2n & 0 & r+l & 0 \\ r-l & 2n & r-l & 0 \\ 0 & t-b & t-b & 0 \\ 0 & 0 & -(f+n) & -2fn \\ 0 & 0 & f-n & f-n \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5/3 & -8/3 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

25

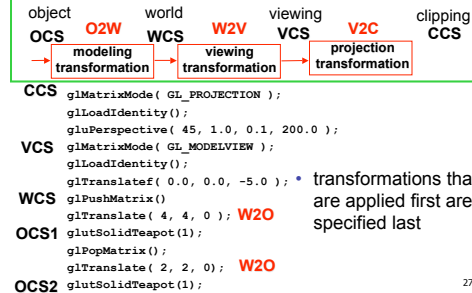
## Perspective Example

$$\begin{bmatrix} 1 & & & \\ & -1 & & \\ & -5z_{VCS}/3 - 8/3 & & \\ & -z_{VCS} & & \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & -5/3 & -8/3 & \\ & -1 & & \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ z_{VCS} \\ 1 \end{bmatrix}$$

$$\frac{1}{w} \rightarrow \begin{aligned} x_{NDCS} &= -1/z_{VCS} \\ y_{NDCS} &= 1/z_{VCS} \\ z_{NDCS} &= \frac{5}{3} + \frac{8}{3z_{VCS}} \end{aligned}$$

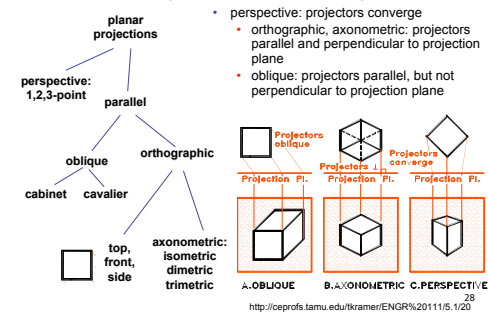
26

## OpenGL Example



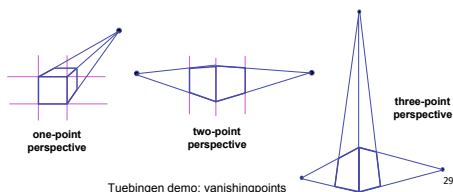
27

## Projection Taxonomy



## Perspective Projections

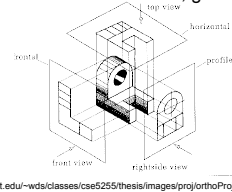
- projectors converge on image plane
- select how many vanishing points
  - one-point: projection plane parallel to two axes
  - two-point: projection plane parallel to one axis
  - three-point: projection plane not parallel to any axis



29

## Orthographic Projections

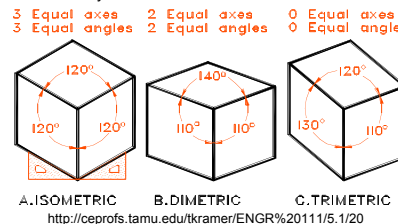
- projectors parallel, perpendicular to image plane
- image plane normal parallel to one of principal axes
- select view: top, front, side
- every view has true dimensions, good for measuring



30

## Axonometric Projections

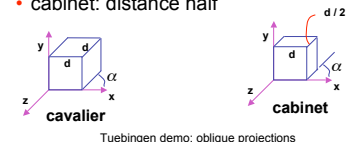
- projectors parallel, perpendicular to image plane
- image plane normal not parallel to axes
- select axis lengths
- can see many sides at once



31

## Oblique Projections

- projectors parallel, oblique to image plane
- select angle between front and z axis
  - lengths remain constant
- both have true front view
  - cavalier: distance true
  - cabinet: distance half



32