Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name: 

Student Number: 

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 1</td>
<td>/ 2</td>
</tr>
<tr>
<td>Question 2</td>
<td>/ 4</td>
</tr>
<tr>
<td>Question 3</td>
<td>/ 4</td>
</tr>
<tr>
<td>Question 4</td>
<td>/ 7</td>
</tr>
<tr>
<td>Question 5</td>
<td>/ 2</td>
</tr>
<tr>
<td>Question 6</td>
<td>/ 3</td>
</tr>
<tr>
<td>Question 7</td>
<td>/ 5</td>
</tr>
<tr>
<td>TOTAL</td>
<td>/ 27</td>
</tr>
</tbody>
</table>

This exam has 7 questions, for a total of 27 points.
1. (2 points) Back-face culling

For the following scene, the polygons forming a closed solid object are represented by edges. Which faces would be removed by back-face culling? Show your work for borderline cases.

2. BSP trees

(a) (2 points) Build a BSP tree for the following scene by inserting the polygons in alphabetical order. In your binary tree, draw the '+' halfspaces to the right, and the '-' halfspaces to the left.

(b) (2 points) Use the BSP tree to determine a back-to-front ordering from the given eye position.
3. (4 points) Sketch the ambient, diffuse, specular, and total illumination for the following scene as a function of $x$. Assume the Phong illumination model, i.e., $I = k_a I_a + k_d I_d (N \cdot L) + k_s I_s (R \cdot V)^n$,
where $k_a = 0.2$, $k_d = 0.6$, $k_s = 0.6$, $I_a = I_d = I_s = 1.0$, $n = 300$. 

---

[Diagram of a light source and an eye with a graph showing the illumination $I(x)$ as a function of $x$.]
4. Scan Conversion

(a) (1 point) Develop an explicit line equation for line $P_1P_2$.

(b) (1 point) Develop an implicit line equation for line $P_1P_2$.

(c) (1 point) Develop a parametric line equation for line $P_1P_2$.

(d) (1 point) Given the barycentric coordinates $P = \alpha P_1 + \beta P_2 + \gamma P_3$, give the value of $\beta$ at each triangle vertex and sketch the line on the triangle where $\beta = 1/3$.

(e) (2 points) Develop an expression for computing $\beta$ as a function of $x$ and $y$ for an arbitrary point $P(x, y)$.

(f) (1 point) Which pixels would be set when scan converting the given triangle? Assume that pixel centres are located at integer coordinates, as shown by example for pixel (2,1). State any additional assumptions that you make.
5. (2 points) Texture Mapping

A texture map with the letters “ABCD” is mapped onto the given triangle. Draw the texture map (i.e., the letters) as they would appear on the texture-mapped triangle.

6. (3 points) Clipping

Clip the line $P_1P_2$ to the plane $F(P)$, given $P_1 = (0, 0, 0)$, $P_2 = (1, 5, 6)$, and $F(P) = x + 2y + 3z - 10$. Assume that $F(P) < 0$ represents the half-space that should be clipped. Give the vertex coordinates of the clipped line.
7. Ray-Triangle Intersection

(a) (2 points) Describe in as much detail as you can how one would compute a ray-triangle intersection test. Hint: Write the equation of a point on the ray in terms of a parametric form, and write the equation of a point in the triangle as a set of Barycentric coordinates. In the end you can obtain a system of 3 equations in 3 unknowns.

(b) (1 point) Describe how one would test whether two 3D triangles intersect each other using the ray-triangle intersection test given above.

(c) (2 points) Suppose we use the triangle-triangle collision detection test as the basis for testing to see whether two solid objects, each represented by a triangulated surface mesh, intersect each other, i.e., whether they are colliding. List several ways how the number of full triangle-to-triangle intersection tests can be minimized, or the tests themselves can be sped up.