

Bidirectional typing

$\Gamma(x) = \text{rat}$ Synth-var
 $\Gamma \vdash (\text{Id } x) \Rightarrow \text{rat}$ Sub-refl
 $\Gamma \vdash (\text{Id } x) \Leftarrow \text{rat}$ Check-sub
 $x:\text{rat}, \Gamma \vdash (\text{Pair } (\text{Id } x) (\text{Id } x)) \Leftarrow \text{rat} \times \text{rat}$ Check-pair
 $\emptyset \vdash (\text{Lam } x \text{ (Pair } (\text{Id } x) (\text{Id } x))) \Leftarrow (\text{rat} \rightarrow (\text{rat} \times \text{rat}))$ Check-lam
 $\Gamma = x:\text{rat}, \Gamma$

$\Gamma(x) = \text{bool}$ Synth-var
 $\Gamma \vdash (\text{Id } x) \Rightarrow \text{bool}$ Sub-refl
 $\Gamma \vdash (\text{Id } x) \Leftarrow \text{bool}$ Check-sub
 $x:\text{bool}, \emptyset \vdash (\text{Pair } (\text{Id } x) (\text{Id } x)) \Leftarrow \text{bool} \times \text{bool}$ Check-pair
 $\emptyset \vdash (\text{Lam } x \text{ (Pair } (\text{Id } x) (\text{Id } x))) \Leftarrow \text{bool} \rightarrow (\text{bool} \times \text{bool})$ Check-lam
 $\Gamma_x = x:\text{bool}, \emptyset$ $A1 = \text{bool}$ $A1 \rightarrow (A1 \times A1)$

$\Gamma(x) = a$ Synth-var $\Gamma_x = x:a, a \text{ type}, \emptyset$
 $\Gamma \vdash (\text{Id } x) \Rightarrow a$ Sub-refl
 $\Gamma \vdash (\text{Id } x) \Leftarrow a$ Check-sub
 $x:a, a \text{ type}, \emptyset \vdash (\text{Pair } (\text{Id } x) (\text{Id } x)) \Leftarrow a \times a$ Check-pair
 $a \text{ type}, \emptyset \vdash (\text{Lam } x \text{ (Pair } (\text{Id } x) (\text{Id } x))) \Leftarrow a \rightarrow (a \times a)$ Check-lam
 $\emptyset \vdash (\text{All } a \text{ (Lam } x \text{ (Pair } (\text{Id } x) (\text{Id } x)))) \Leftarrow \forall a. (a \rightarrow (a \times a))$ Check-all
 $\emptyset \vdash (\text{Anno } e_{\text{All}} \text{ (}\forall a. (a \rightarrow (a \times a))\text{)}) \Rightarrow (\forall a. (a \rightarrow (a \times a)))$ Synth-anno
 $\emptyset \vdash e_{\text{Anno}} \Rightarrow \forall a. (a \rightarrow (a \times a))$ Synth-at
 $\emptyset \vdash e_{\text{All}} \Rightarrow \text{pos} \rightarrow (\text{pos} \times \text{pos})$ Synth-app
 $\emptyset \vdash (\text{App } e_{\text{All}} \text{ (Num 2)}) \Rightarrow (\text{pos} \times \text{pos})$
 $A = \text{pos}$
 $= [\text{pos}/a] (a \rightarrow (a \times a))$
 $= (\text{pos} \rightarrow (\text{pos} \times \text{pos}))$