CPSC 311 2016W1
Review 2016-10-24

$$\dfrac{\dfrac{\dfrac{}{(\text{Bfalse}) \Downarrow (\text{Bfalse})}\ \text{Eval-bfalse} \qquad \dfrac{}{(\text{Num } 2) \Downarrow (\text{Num } 2)}\ \text{Eval-num}}{(\text{Ite } (\text{Bfalse}) \ (\text{Num } 1) \ (\text{Num } 2)) \Downarrow (\text{Num } 2)}\ \text{Eval-ite-false} \qquad e\text{Ite} \Downarrow (\text{Num } 2)}{(\text{Pair } \underbrace{(\text{Ite } (\text{Bfalse}) \ (\text{Num } 1) \ (\text{Num } 2))}_{e\text{Ite}} \ \underbrace{(\text{Ite } (\text{Bfalse}) \ (\text{Num } 1) \ (\text{Num } 2))}_{e1 = e\text{Ite}}) \Downarrow \underbrace{(\text{Pair } (\text{Num } 2) \ (\text{Num } 2))}_{e\text{Pair} \ \ v\text{Pair}}}\ \text{Eval-pair}$$

$$\dfrac{\dfrac{\dfrac{}{e1 \Downarrow (\text{Lam } y \text{ num } (\text{Pair } (\text{Id } y) \ (\text{Id } y)))}\ \text{Eval-lam} \qquad = \qquad [e2/y](\text{Pair } (\text{Id } y) \ (\text{Id } y)) \Downarrow (\text{Pair } e2 \ e2)}{(\text{App } \underbrace{\boxed{(\text{Lam } y \text{ num } (\text{Pair } (\text{Id } y) \ (\text{Id } y)))}}_{e1} \ \underbrace{(\text{Ite } (\text{Bfalse}) \ (\text{Num } 1) \ (\text{Num } 2))}_{e2}) \Downarrow e\text{Pair} \ \ v\text{Pair}}\ \text{Eval-app-expr}}{}\ v\text{Pair} \ e\text{Pair}$$

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$$\dfrac{\Gamma(g) = \overbrace{(num * num) \to bool}^{A \to B} \qquad \dfrac{\dfrac{}{\Gamma \vdash (Num\ 1) : num}\ \text{Type-num} \qquad \dfrac{}{\Gamma \vdash (Num\ 2) : num}\ \text{Type-num}}{\Gamma \vdash (Pair\ (Num\ 1)\ (Num\ 2)) : num * num}\ \text{Type-pair}}{\underbrace{\Gamma \vdash (Id\ g) : (num * num) \to bool}_{\text{Type-var}}}$$

$$\dfrac{}{\underbrace{g : (num * num) \to bool,\ \emptyset}_{\Gamma} \vdash (App\ \underbrace{(Id\ g)}_{e1}\ \underbrace{(Pair\ (Num\ 1)\ (Num\ 2))}_{e2}) : bool}\ \text{Type-app}$$

$A = num * num$
$B = bool$

Label typing contexts to avoid writing them out in every premise

You can also label expressions. But don't mix different e1s or e2s in the same derivation.

$$\dfrac{\dfrac{}{\emptyset \vdash (Num\ 1) : num}\ \text{Type-num} \qquad z : num, \emptyset \vdash (Id\ z) : num}{\emptyset \vdash (Let\ \underbrace{z}_{x}\ \underbrace{(Num\ 1)}_{e}\ \underbrace{(Id\ z)}_{eBody}) : num}$$

$$\dfrac{\dfrac{}{\Gamma_{xy} \vdash (Num\ 1) : num}\ \text{Type-num} \qquad z : num, \Gamma_{xy} \vdash (Id\ z) : num}{\underbrace{x : num,\ y : bool,\ \emptyset}_{\Gamma_{xy}} \vdash (Let\ z\ (Num\ 1)\ (Id\ z)) :}$$

$\Gamma_{xyz} \vdash (Id\ z) : num$

$\Gamma_{xyz} = z : num, \Gamma_{xy}$