# "Expanding" [typing] rules

$A1 * A2$

(let ([Ae (typeof tc e)]) ... type-case Type Ae [T*(A1 A2) ...) (let ([B (typeof tc-ext eBody)]) B)

$$\frac{\Gamma \vdash e : A1*A2 \qquad x1:A1, x2:A2, \Gamma \vdash eBody : B}{\Gamma \vdash (\text{Pair-case } e\ x1\ x2\ eBody) : B}\ \text{Type-pair-case}$$

(typeof tc ... )

$x1:A1, x2:A2, \Gamma$

B "free" or "a free output"

B can be any type here. We don't need to inspect it in the branch for Pair-case.

Can we change the rule so the type in the first premise is free?

$$\frac{\Gamma \vdash e : Ae \quad Ae = A1*A2 \qquad x1:A1, x2:A2, \Gamma \vdash eBody : B}{\Gamma \vdash (\text{Pair-case } e\ x1\ x2\ eBody) : B}$$

NOT free: must be a →

NOT free: Must match the A from the first premise

$$\frac{\Gamma \vdash e1 : A \to B \qquad \Gamma \vdash e2 : A}{\Gamma \vdash (\text{App } e1\ e2) : B}\ \text{Type-app}$$

$$\frac{(\Gamma \vdash e1 : Ae1 \quad Ae1 = A \to B) \qquad (\Gamma \vdash e2 : Ae2 \quad Ae2 = A)}{\Gamma \vdash (\text{App } e1\ e2) : B}$$