CPSC 311: Small-step semantics: Rules for smallstep.rkt **(DRAFT)** ("lec-smallstep-2")

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October 30, 2016

To define what $\mathcal{C}$ means, we'll use a BNF grammar. We've been using BNFs to define the concrete syntax of languages, but BNFs are versatile and can also be used with *abstract* syntax. To (hopefully) clarify that this BNF is describing abstract syntax, not concrete syntax, I'll follow the convention I've been using in the rules, where we write e, v, n, etc. rather than using angle brackets $\langle \mathtt{E} \rangle$.

This is also an opportunity to define values v using a BNF:

$$\begin{array}{rrcl} \text{Values} & v & ::= & (\mathsf{Num}\ n) \\ & & \mid & (\mathsf{Lam}\ x\ e) \\ & & \mid & (\mathsf{Btrue}) \\ & & \mid & (\mathsf{Bfalse}) \end{array}$$

Now, the definition of evaluation contexts:

$$\begin{array}{rrcl} \text{Evaluation contexts} & \mathcal{C} & ::= & [\,] \\ & & \mid & (\mathsf{Binop}\ \mathrm{op}\ \mathcal{C}\ e) \\ & & \mid & (\mathsf{Binop}\ \mathrm{op}\ v\ \mathcal{C}) \\ & & \mid & (\mathsf{App}\ \mathcal{C}\ e) \\ & & \mid & (\mathsf{App}\ v\ \mathcal{C}) \\ & & \mid & (\mathsf{Let}\ x\ \mathcal{C}\ e) \\ & & \mid & (\mathsf{Ite}\ \mathcal{C}\ e\ e) \end{array}$$

The empty brackets $[\,]$ are called a "hole". Some examples of evaluation contexts:

$(\mathsf{App}\ [\,]\ (\mathsf{App}\ e3\ e4))$
$(\mathsf{Binop}\ (\mathsf{Minusop})\ (\mathsf{Num}\ 5)\ [\,])$
$(\mathsf{App}\ (\mathsf{App}\ [\,]\ e1)\ e2))$

$\boxed{e1 \longrightarrow e2}$ Expression $e1$ steps to $e2$

Reduction rules:

$$\frac{v1 \ \mathsf{op} \ v2 = v}{(\mathsf{Binop} \ \mathsf{op} \ v1 \ v2) \longrightarrow v} \ \text{Step-binop} \qquad \frac{}{(\mathsf{App} \ (\mathsf{Lam} \ x \ eB) \ v) \longrightarrow [v/x]eB} \ \text{Step-app-value}$$

$$\frac{}{(\mathsf{Let} \ x \ v1 \ e2) \longrightarrow [v1/x]e2} \ \text{Step-let}$$

$$\frac{}{(\mathsf{Ite} \ (\mathsf{Btrue}) \ e\mathsf{Then} \ e\mathsf{Else}) \longrightarrow e\mathsf{Then}} \ \text{Step-ite-true}$$

$$\frac{}{(\mathsf{Ite} \ (\mathsf{Bfalse}) \ e\mathsf{Then} \ e\mathsf{Else}) \longrightarrow e\mathsf{Else}} \ \text{Step-ite-false}$$

$$\frac{}{(\mathsf{Rec} \ u \ e) \longrightarrow [(\mathsf{Rec} \ u \ e)/u]e} \ \text{Step-rec}$$

Context rule:

$$\frac{e \longrightarrow e'}{\mathcal{C}[e] \longrightarrow \mathcal{C}[e']} \ \text{Step-context}$$

$\boxed{e \ \text{free-variable-error}}$ Trying to step e encounters a free variable

$$\frac{}{\mathcal{C}[(\mathsf{Id} \ x)] \ \text{free-variable-error}} \ \text{FVerr-context}$$

Figure 1 Small-step semantics