

Question 1 [55 points]: "If it's 'dynamic', it *must* be better."

The following rules define an environment-based semantics for lexically-scoped functions, lam, and dynamically-scoped functions, ds-lam.

$\boxed{\text{env} \vdash e \Downarrow v}$ Under environment env , expression e evaluates to value v

$$\frac{}{\text{env} \vdash (\text{num } n) \Downarrow (\text{num } n)} \text{Env-num} \quad \frac{\text{env} \vdash e1 \Downarrow (\text{num } n1) \quad \text{env} \vdash e2 \Downarrow (\text{num } n2)}{\text{env} \vdash (\text{add } e1 \ e2) \Downarrow (\text{num } n1+n2)} \text{Env-add}$$

$$\frac{\text{env} \vdash e1 \Downarrow v1 \quad x=v1, \text{env} \vdash e2 \Downarrow v2}{\text{env} \vdash (\text{with } x \ e1 \ e2) \Downarrow v2} \text{Env-with}$$

$$\frac{\text{lookup}(\text{env}, x) = e}{\text{env} \vdash (\text{id } x) \Downarrow e} \text{Env-id}$$

$$\frac{\text{lookup}(\text{env}, x) \text{ undefined}}{\text{env} \vdash (\text{id } x) \text{ unknown-id-error}} \text{Env-unknown-id}$$

$$\frac{}{\text{env} \vdash (\text{lam } x \ e1) \Downarrow (\text{clo } \text{env} \ (\text{lam } x \ e1))} \text{Env-lam} \quad \frac{}{\text{env} \vdash (\text{clo } \text{env}_{\text{old}} \ e) \Downarrow (\text{clo } \text{env}_{\text{old}} \ e)} \text{Env-clo}$$

$$\frac{\text{env} \vdash e1 \Downarrow (\text{clo } \text{env}_{\text{old}} \ (\text{lam } x \ eB)) \quad \text{env} \vdash e2 \Downarrow v2 \quad x=v2, \text{env}_{\text{old}} \vdash eB \Downarrow v}{\text{env} \vdash (\text{app } e1 \ e2) \Downarrow v} \text{Env-app}$$

$$\frac{}{\text{env} \vdash (\text{ds-lam } x \ e1) \Downarrow (\text{ds-lam } x \ e1)} \text{Env-ds-lam} \quad \frac{\text{env} \vdash e1 \Downarrow (\text{ds-lam } x \ eB) \quad \text{env} \vdash e2 \Downarrow v2 \quad x=v2, \text{env} \vdash eB \Downarrow v}{\text{env} \vdash (\text{app } e1 \ e2) \Downarrow v} \text{Env-ds-app}$$

Assume that $\text{lookup}(\text{env}, x)$ returns the **leftmost** binding of x . For example:

$$\text{lookup}((x=(\text{num } 2), x=(\text{num } 1), \emptyset), x) = (\text{num } 2)$$

Consider the following expression, shown in concrete syntax (left) and in abstract syntax (right).

```
{with {y 100}
  {with {f {with {y 10}
    {lam x {+ x y}}}}
    {with {y 2}
      {app f y}}}}
```

```
(with y (num 100)
  (with f (with y (num 10)
    (lam x (add (id x) (id y))))
    (with y (num 2)
      (app (id f) (id y))))))
```

Q1a [10 points] Complete the **abstract syntax tree** for the above expression.

\Rightarrow

