

Review

2015-11-30

①

closures/
environments/
mutable state
dynamic scope

subtyping/
type inference/
bidirectional typing

Fig. 9

$(\text{num } n) \Downarrow (\text{num } n)$ Eval-num

$\frac{\text{Eval-num}}{(\text{num } 1) \Downarrow (\text{num } 1)}$ rule with no premises

$\frac{(\text{num } 1) \Downarrow (\text{num } 1) \quad (\text{num } 1) \Downarrow (\text{num } 1) \quad \text{Eval-add}}{(\text{add } (\text{num } 1) (\text{num } 1)) \Downarrow (\text{num } 2)}$ in complete

$\frac{(\text{num } 1) \Downarrow (\text{num } 1) \quad (\text{num } 1) \Downarrow (\text{num } 1)}{(\text{add } (\text{num } 1) (\text{num } 1)) \Downarrow (\text{num } 2)}$ complete

$\frac{}{(\text{num } 1) \Downarrow \underline{v1} \text{?}}$ Eval-num

$\frac{(\text{num } 1) \Downarrow (\text{num } 1) \quad (\text{num } 1) \Downarrow (\text{num } 1) \quad \text{Eval-num}}{(\text{add } (\text{num } 1) (\text{num } 1)) \Downarrow (\text{num } 2)}$ Eval-num complete

$\frac{}{(\text{num } 1) \Downarrow \frac{(\text{num } 1)}{v1}}$ Eval-num

Derivation trees

$$\frac{\Delta \vdash P \quad \Delta \vdash Q}{\Delta \vdash P \wedge Q} \wedge \text{Intro} \qquad \frac{\Delta \vdash P}{\Delta \vdash P \vee Q} \vee \text{Intro1} \qquad \frac{\Delta \vdash Q}{\Delta \vdash P \vee Q} \vee \text{Intro2}$$

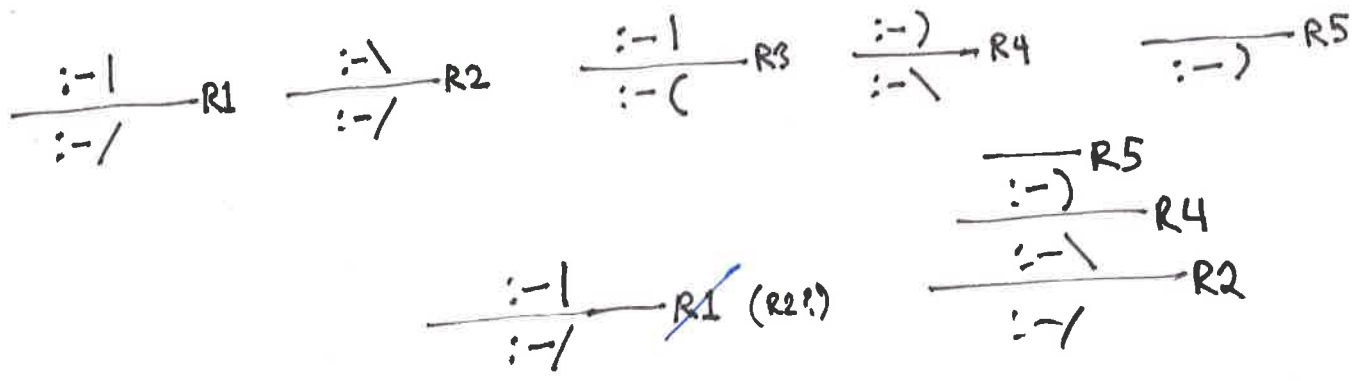
$$\frac{\Delta \vdash P \wedge Q}{\Delta \vdash P} \wedge \text{Elim1} \qquad \frac{\Delta \vdash P \wedge Q}{\Delta \vdash Q} \wedge \text{Elim2} \qquad \frac{\Delta \vdash P \vee Q \quad \Delta, P \vdash R \quad \Delta, Q \vdash R}{\Delta \vdash R} \vee \text{Elim}$$

$$\frac{P \in \Delta}{\Delta \vdash P} \text{ASSUM}$$

Props.

$\Delta ::= \emptyset$
 $\quad \quad \quad | P, \Delta$

$P ::= a | b | c | d | \dots$
 $Q ::= | P \wedge P$ conjunction and
 $R ::= | P \vee P$ disjunction or



$$\frac{\frac{a \in a, b, \emptyset}{\wedge \text{Intro } a, b, \emptyset \vdash a} \text{assum} \quad \frac{b \in a, b, \emptyset}{a, b, \emptyset \vdash b} \text{assum}}{a, b, \emptyset \vdash a \wedge b} \wedge \text{Intro}$$

$\underbrace{\quad}_{\Delta}$ $\underbrace{\quad}_{P}$ $\underbrace{\quad}_{Q}$

$$\frac{\frac{a \wedge b \in a \wedge b}{a \wedge b \vdash a \wedge b} \text{assum} \quad \frac{a \wedge b \vdash a \wedge b}{a \wedge b \vdash b} \wedge \text{Elim2} \quad \frac{a \wedge b \vdash a \wedge b}{a \wedge b \vdash a} \wedge \text{Elim1}}{a \wedge b \vdash b \wedge a} \wedge \text{Intro}$$

$\underbrace{\quad}_{P}$ $\underbrace{\quad}_{Q}$

$$\frac{\frac{\frac{a, b, \emptyset \vdash a}{a, b, \emptyset \vdash a} \text{assum} \quad \frac{a, b, \emptyset \vdash a}{a, b, \emptyset \vdash a} \text{assum}}{a, b, \emptyset \vdash a \wedge Qa} \wedge \text{Intro}}{a, b, \emptyset \vdash a} \wedge \text{Elim1}$$

$$\frac{b \wedge a \in a \wedge b}{a \wedge b \vdash b \wedge a} \text{assum}$$

$$\frac{\frac{\frac{a \vee b \vdash a \vee b}{a \vee b \vdash a \vee b} \text{assum} \quad \frac{\frac{a \vee b, a \vdash a}{a \vee b, a \vdash b \vee a} \text{assum} \quad \frac{a \vee b, b \vdash b}{a \vee b, b \vdash b \vee a} \text{assum}}{a \vee b, a \vdash b \vee a} \vee \text{Intro2} \quad \frac{a \vee b, b \vdash b \vee a}{a \vee b, b \vdash b \vee a} \vee \text{Intro1}}{a \vee b \vdash b \vee a} \vee \text{Elim}$$

$\underbrace{\quad}_{R}$

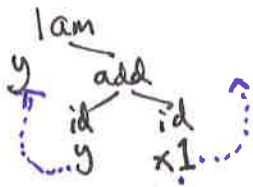
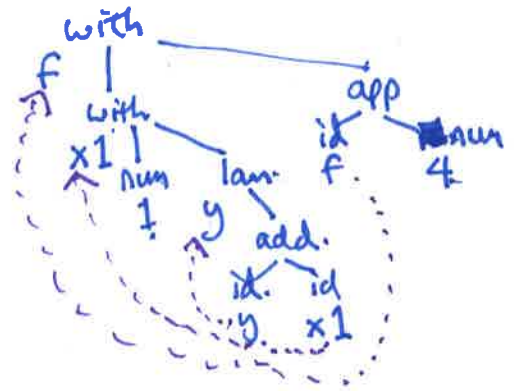
incomplete derivations;
can't show
 $a \vee b \vdash a$
 $a \vee b \vdash b$

$$\frac{a \vee b \vdash a \quad a \vee b \vdash b}{a \vee b \vdash a \wedge b} \wedge \text{Intro}$$

SCOPE (LEXICAL), CLOSURES

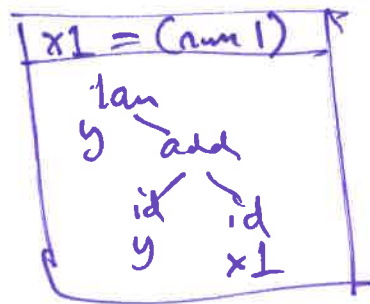
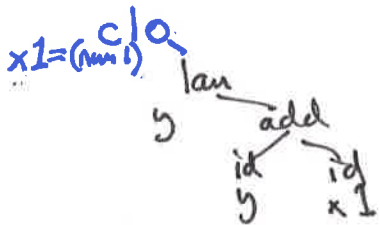
(with f (with x1 (num 1)
 (lam y (add (id y) (id x1)))) add 1 to y
 (app (id f) (num 4))
)

$x1 = (num\ 1), \emptyset \vdash (lam\ y\ \dots) \Downarrow (lam\ y\ \dots)$



~~f = (lam~~

$f = (lam\ y\ (add\ (id\ y)\ (id\ x1))), \emptyset \vdash (app\ (id\ f)\ (num\ 4)) \Downarrow$



(flip)

- Concrete syntax:

$$\langle E \rangle ::= \dots \mid \{ \text{flip } \langle id \rangle \langle E \rangle \}$$

"flip x in e"

- Abstract syntax:

$$e ::= \dots \mid (\text{flip } x \ e)$$

$$v ::= \dots$$

(flip NOT a value)

define-type:

(define-type E

[flip (x symbol?) (body E?)])

)

- Substitution:

$$\begin{aligned} \text{subst}(\text{flip } x \ eB, x, e2) &= (\text{flip } x \ eB) \\ \text{subst}(\text{flip } y \ eB, x, e2) &= (\text{flip } y \ \text{subst}(eB, x, e2)) \\ &\text{if } y \neq x \end{aligned}$$

- Dynamic semantics

1. Substitution-based, big-step

$$\text{Eval-flip-1} \frac{e \Downarrow v \quad \text{subst}(eB, x, (\text{btrue})) \Downarrow v}{(\text{flip } x \ eB) \Downarrow v} \quad \text{Eval-flip-0} \frac{\text{subst}(eB, x, (\text{bfalse})) \Downarrow v}{(\text{flip } x \ eB) \Downarrow v}$$

2. Environment-based, big-step

$$\text{Env-flip-1} \frac{\text{env} \vdash e \Downarrow v \quad x = (\text{btrue}), \text{env} \vdash eB \Downarrow v}{\text{env} \vdash (\text{flip } x \ eB) \Downarrow v} \quad \text{Env-flip-0} \frac{x = (\text{bfalse}), \text{env} \vdash eB \Downarrow v}{\text{env} \vdash (\text{flip } x \ eB) \Downarrow v}$$

3. ... with a store

$$\text{SEnv-flip-1} \frac{\text{env}; S1 \vdash e \Downarrow v; S2 \quad x = (\text{btrue}), \text{env}; S1 \vdash eB \Downarrow v; S2}{\text{env}; \overset{S1}{S2} \vdash (\text{flip } x \ eB) \Downarrow v; S2} \quad \text{SEnv-flip-0 similar}$$

- Static semantics

-Typing:

$$\frac{x: \text{Bool}, \Gamma \vdash eB : B}{\Gamma \vdash (\text{flip } x \ eB) : B}$$

$$\frac{}{(bfalse) \Downarrow (bfalse)} \text{Eval-false}$$

$$\begin{aligned} & \frac{}{} = \\ & \frac{\frac{(bfalse)}{\text{subst}((id\ x),\ x,\ (bfalse)) \Downarrow (bfalse)} \text{Eval-false}}{(flip\ x\ (id\ x)) \Downarrow (bfalse)} \text{Eval-flip-0} \end{aligned}$$