Review

2015-11-30 (1)

closures / environments / mutable state

dynamic scope

subtyping / type inference / bidirectional typing

$$\frac{}{(\text{num } n) \Downarrow (\text{num } n)}\ \text{Eval-num}$$

Eval-num $\frac{}{(\text{num } 1) \Downarrow (\text{num } 1)}$ rule with no premises

$$\frac{(\text{num } 1) \Downarrow (\text{num } 1) \quad (\text{num } 1) \Downarrow (\text{num } 1)}{(\text{add } (\text{num } 1)\ (\text{num } 1)) \Downarrow (\text{num } 2)}\ \text{Eval-add}$$ incomplete

$$\frac{(\text{num } 1) \Downarrow (\text{num } 1) \quad (\text{num } 1) \Downarrow (\text{num } 1)}{(\text{add } (\text{num } 1)\ (\text{num } 1)) \Downarrow (\text{num } 2)}$$ complete

$$\frac{\dfrac{}{(\text{num } 1) \Downarrow (\text{num } 1)}\ \text{Eval-num} \quad \dfrac{}{(\text{num } 1) \Downarrow (\text{num } 1)}\ \text{Eval-num}}{(\text{add } (\text{num } 1)\ (\text{num } 1)) \Downarrow (\text{num } 2)}$$ complete

$$\frac{}{(\text{num } 1) \Downarrow v1 \stackrel{?}{=}}\ \text{Eval-num}$$

$$\frac{}{(\text{num } 1) \Downarrow \underbrace{(\text{num } 1)}_{v1}}\ \text{Eval-num}$$

Derivation trees

$$\frac{\Delta \vdash P \quad \Delta \vdash Q}{\Delta \vdash P \wedge Q}\ \wedge\text{Intro} \qquad \frac{\Delta \vdash P}{\Delta \vdash P \vee Q}\ \vee\text{Intro1} \qquad \frac{\Delta \vdash Q}{\Delta \vdash P \vee Q}\ \vee\text{Intro2}$$

$$\frac{\Delta \vdash P \wedge Q}{\Delta \vdash P}\ \wedge\text{Elim1} \qquad \frac{\Delta \vdash P \wedge Q}{\Delta \vdash Q}\ \wedge\text{Elim2} \qquad \frac{\Delta \vdash P \vee Q \quad \Delta, P \vdash R \quad \Delta, Q \vdash R}{\Delta \vdash R}\ \vee\text{Elim}$$

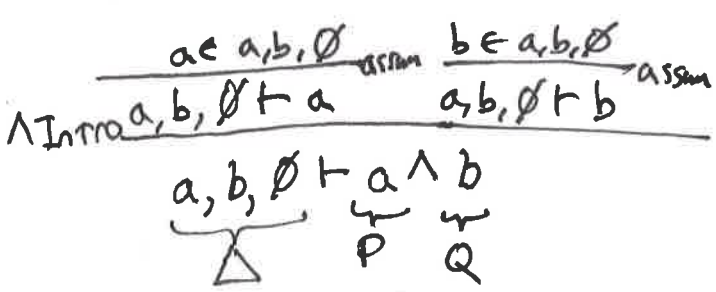
$$\frac{P \in \Delta}{\Delta \vdash P}\ \text{assum}$$

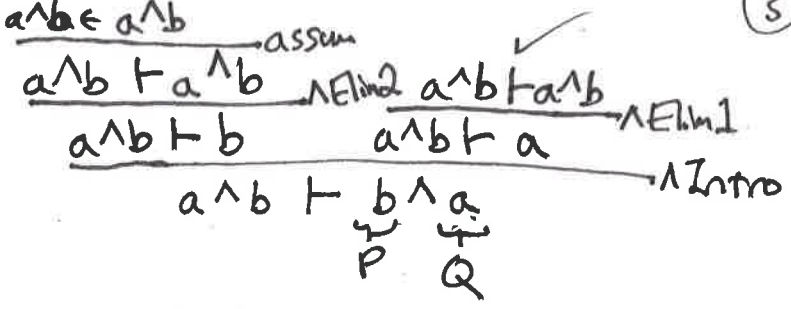
$\Delta ::= \emptyset \mid P, \Delta$

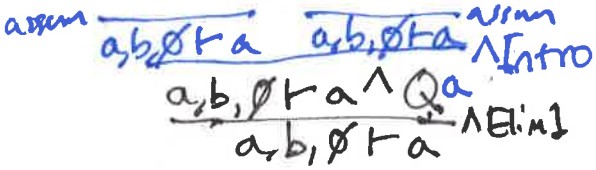
Props. $P, Q, R ::= a \mid b \mid c \mid d \mid \ldots \mid P \wedge P$ conjunction and $\mid P \vee P$ disjunction or

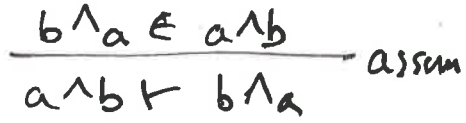
$$\frac{}{\text{:-|}}\ \text{R1} \qquad \frac{}{\text{:-\textbackslash}}\ \text{R2} \qquad \frac{\text{:-|}}{\text{:-(}}\ \text{R3} \qquad \frac{\text{:-)}}{\text{:-\textbackslash}}\ \text{R4} \qquad \frac{}{\text{:-)}}\ \text{R5}$$

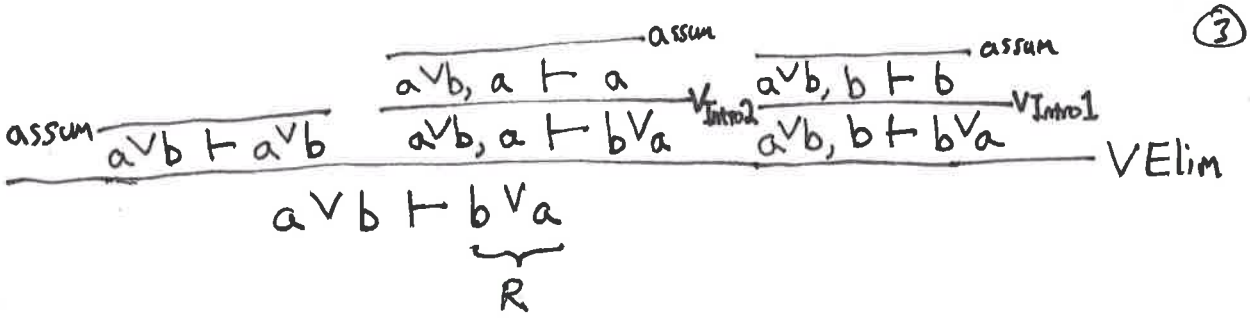
$$\frac{\text{:-|}}{\text{:-/}}\ \text{R1 (R2?)} \qquad \frac{\dfrac{\dfrac{}{\text{:-)}}\ \text{R5}}{\text{:-\textbackslash}}\ \text{R4}}{\text{:-/}}\ \text{R2}$$







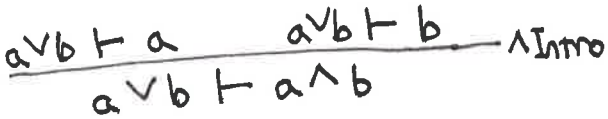




incomplete derivations; can't show

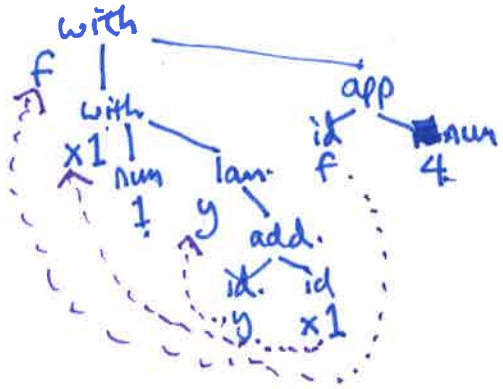
$a \vee b \vdash a$

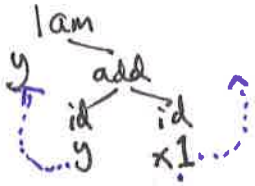
$a \vee b \vdash b$



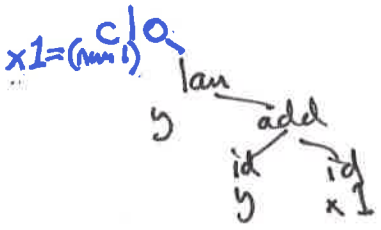
```
(with f (with x1 (num 1)
                 (lam y (add (id y) (id x1))))   add 1 to y
        (app (id f) (num 4))
)
```

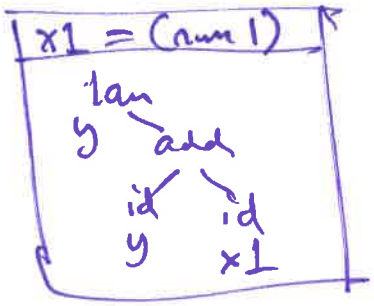
$x1=(num\ 1), \emptyset \vdash (lam\ y \cdots) \Downarrow (lam\ y \cdots)$





$f=(lam\ y\ (add\ (id\ y)\ (id\ x1))), \emptyset \vdash (app\ (id\ f)\ (num\ 4)) \Downarrow$





(flip)

- Concrete syntax:

  ⟨E⟩ ::= ...
  | {flip ⟨id⟩ ⟨E⟩}

  "flip x in e"

- Abstract syntax:

  e ::= ...
  | (flip x e)

  v ::= ...

  (flip NOT a value)

```
define-type:
(define-type E
  [flip (x symbol?) (body E?)]
)
```

- Substitution:

  subst((flip x eB), x, e2) = (flip x eB)
  subst((flip y eB), x, e2) = (flip y subst(eB, x, e2))
  if y≠x

- Dynamic semantics

  1. Substitution-based, big-step

$$\text{Eval-flip-1}\ \frac{e \Downarrow v \qquad \text{subst}(eB, x, (\text{btrue})) \Downarrow v'}{(\text{flip } x\ eB) \Downarrow v} \qquad \text{Eval-flip-0}\ \frac{\text{subst}(eB, x, (\text{bfalse})) \Downarrow v}{(\text{flip } x\ eB) \Downarrow v}$$

  2. Environment-based, big-step

$$\text{Env-flip-1}\ \frac{x=(\text{btrue}), env \vdash eB \Downarrow v}{env \vdash (\text{flip } x\ eB) \Downarrow v} \qquad \text{Env-flip-0}\ \frac{x=(\text{bfalse}), env \vdash eB \Downarrow v}{env \vdash (\text{flip } x\ eB) \Downarrow v}$$

  3. ... with a store

$$\text{SEnv-flip-1}\ \frac{x=(\text{btrue}), env; S1 \vdash eB \Downarrow v; S2}{env; S1 \vdash (\text{flip } x\ eB) \Downarrow v; S2}$$

  SEnv-flip-0 similar

- Static semantics
  - Typing:

$$\frac{x : \text{Bool}, \Gamma \vdash eB : B}{\Gamma \vdash (\text{flip } x\ eB) : B}$$

$$\frac{}{(\text{bfalse}) \Downarrow (\text{bfalse})}\ \text{Eval-false}$$

$$\frac{\overset{(\text{bfalse})}{=:} \quad \frac{}{\text{subst}((\text{id } x),\ x,\ (\text{bfalse})) \Downarrow (\text{bfalse})}\ \text{Eval-false.}}{(\text{flip } x\ (\text{id } x)) \Downarrow (\text{bfalse})}\ \text{Eval-flip-O}$$