

- 2 steps: 1. Identify when to return a particular right-hand side.
 2. Return it: • Gather missing information, if any.
 • Use the information you have.

§4 Evaluation semantics

4.3 Substitution

$$\text{subst}(\text{arguments}) = \text{result}$$

Substitution for Fun++ abstract syntax

$$\begin{aligned} \text{subst}(\text{num } n, x, e2) &= (\text{num } n) \\ \text{subst}(\text{id } x, x, e2) &= e2 \\ \text{subst}(\text{id } y, x, e2) &= (\text{id } y) \quad \text{if } x \neq y \\ \text{subst}(\text{lam } x \text{ eB}, x, e2) &= (\text{lam } x \text{ eB}) \\ \text{subst}(\text{lam } y \text{ eB}, x, e2) &= (\text{lam } y \text{ subst}(\text{eB}, x, e2)) \\ &\quad \text{if } x \neq y \end{aligned}$$

$$\text{subst}(\text{app eFun eArg}, x, e2) = (\text{app } \text{subst}(\text{eFun}, x, e2) \text{ subst}(\text{eArg}, x, e2))$$

$$\text{subst}(\text{binop op eL eR}, x, e2) = (\text{binop op } \text{subst}(\text{eL}, x, e2) \text{ subst}(\text{eR}, x, e2))$$

$$\text{subst}(\text{pair eL eR}, x, e2) = (\text{pair } \text{subst}(\text{eL}, x, e2) \text{ subst}(\text{eR}, x, e2)) \quad \text{show alternate see next page}$$

$$\text{subst}(\text{bfalse}, x, e2) = (\text{bfalse})$$

$$\text{subst}(\text{btrue}, x, e2) = (\text{btrue})$$

$$\text{subst}(\text{ite e eThen eElse}, x, e2) = (\text{ite } \text{subst}(\text{e}, x, e2) \text{ subst}(\text{eThen}, x, e2) \text{ subst}(\text{eElse}, x, e2))$$

$$\begin{aligned} \text{subst}(\text{with } x \text{ e eB}, x, e2) &= (\text{with } x \text{ subst}(\text{e}, x, e2) \text{ eB}) \\ \text{subst}(\text{with } y \text{ e eB}, x, e2) &= (\text{with } y \text{ subst}(\text{e}, x, e2) \text{ subst}(\text{eB}, x, e2)) \\ &\quad \text{if } x \neq y \end{aligned}$$

$$\begin{aligned} \text{subst}(\text{with}^* () \text{ eB}, x, e2) &= (\text{with}^* () \text{ subst}(\text{eB}, x, e2)) \\ \text{subst}(\text{with}^* ((x \text{ e}) \text{ bindings}) \text{ eB}, x, e2) &= (\text{with}^* ((x \text{ subst}(\text{e}, x, e2)) \text{ bindings}) \text{ eB}) \\ \text{subst}(\text{with}^* ((y \text{ e}) \text{ bindings}) \text{ eB}, x, e2) &= (\text{with}^* ((y \text{ subst}(\text{e}, x, e2)) \text{ bindings}') \text{ eB}') \\ &\quad \text{if } x \neq y \\ &\quad \text{and } \text{subst}(\text{with}^* \text{ bindings } \text{ eB}, x, e2) = (\text{with}^* \text{ bindings}' \text{ eB}') \end{aligned}$$

where

$$\text{subst}(\text{pair-case } e \text{ x1 } x2 \text{ eB}, x, e2) = (\text{pair-case } \text{subst}(\text{e}, x, e2) \text{ x1 } x2 \text{ eB})$$

if $x = x1$ or $x = x2$

$$\text{subst}(\text{pair-case } e \text{ x1 } x2 \text{ eB}, x, e2) = (\text{pair-case } \text{subst}(\text{e}, x, e2) \text{ x1 } x2 \text{ subst}(\text{eB}, x, e2))$$

if $x \neq x1$ and $x \neq x2$

$$\text{subst}(\text{rec } x \text{ eB}, x, e2) = (\text{rec } x \text{ eB})$$

$$\text{subst}(\text{rec } y \text{ eB}, x, e2) = (\text{rec } y \text{ subst}(\text{eB}, x, e2))$$

if $x \neq y$

Alternate:

$$\text{Suppose } \text{subst}((\cdot; \cdot; \cdot)) = (\text{with}^* \text{ bindings}' \text{ eB}') \quad *$$

$$\text{Return } (\text{with}^* ((y \text{ subst}(\text{e}, x, e2)) \text{ bindings}') \text{ eB}') \quad *$$

* What if subst returns something else?

$$\text{subst}(\text{pair } e_L e_R, x, e_2) = (\text{pair } e_L' e_R')$$

where
 $\text{subst}(e_L, x, e_2) = e_L'$
and $\text{subst}(e_R, x, e_2) = e_R'$

(alternate:

Suppose

$\text{subst}(e_L, x, e_2) = e_L'$
and $\text{subst}(e_R, x, e_2) = e_R'$.

Return

$(\text{pair } e_L' e_R').$)

(let*([eL-subst (subst eL x e2)]
[eR-subst (subst eR x e2)])

(pair eL-subst
eR-subst))