Personal Research Areas

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ONR Computational Methods for Collaborative Control MURI (N00014-02-1-0729)

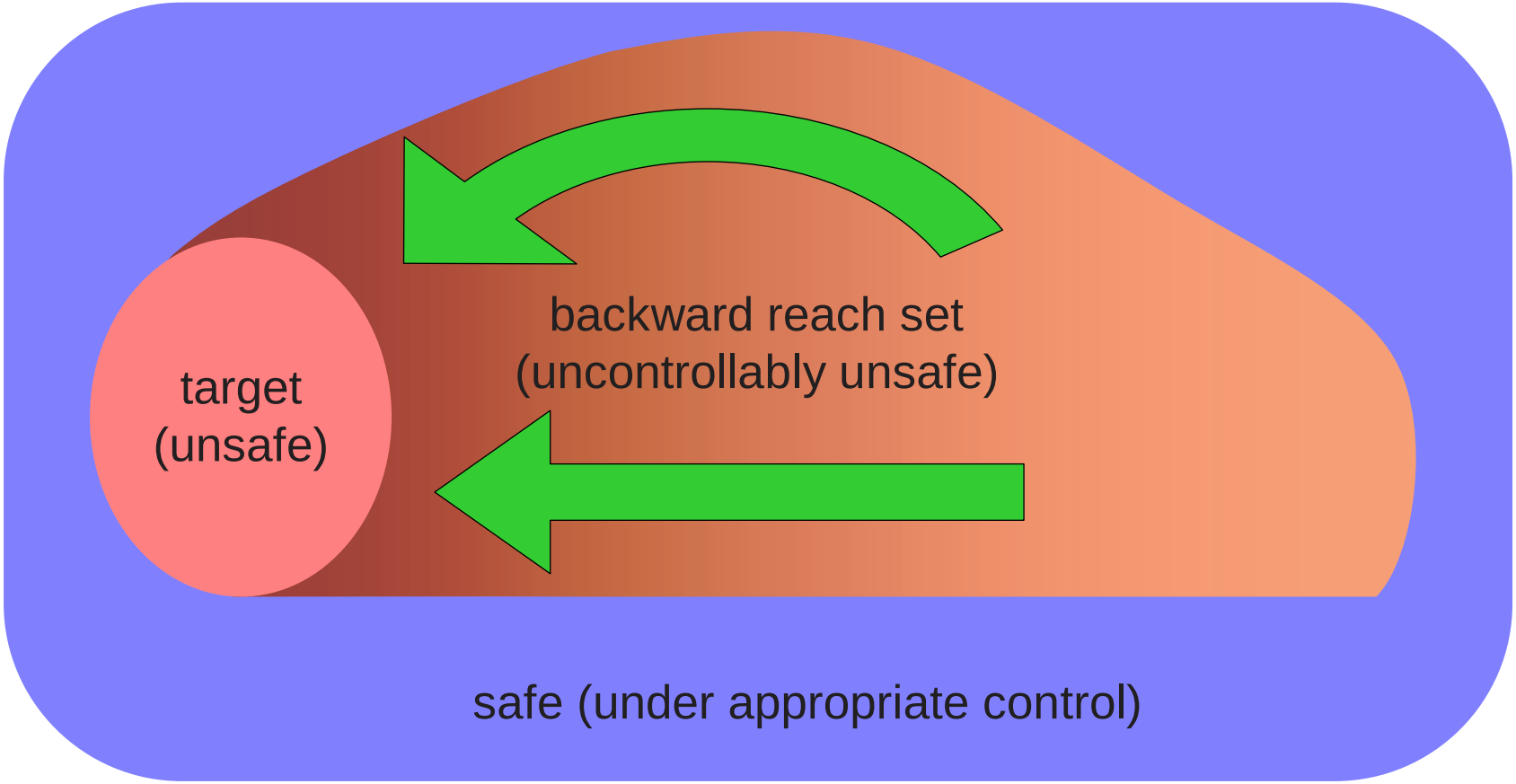
Outline

- Reach sets for control verification
  - Filtering pilot commands for safety
- Dynamic surfaces
  - Particle level set
  - Free surface fluid simulation
- Robotic path planning
  - Shortest continuous path, subject to constraints
  - Optimality for multiple vehicle and/or various norms
- Hamilton-Jacobi PDEs
  - The common link

- Just for fun: **none** of this material is from the course
  - It is graduate level and/or research material

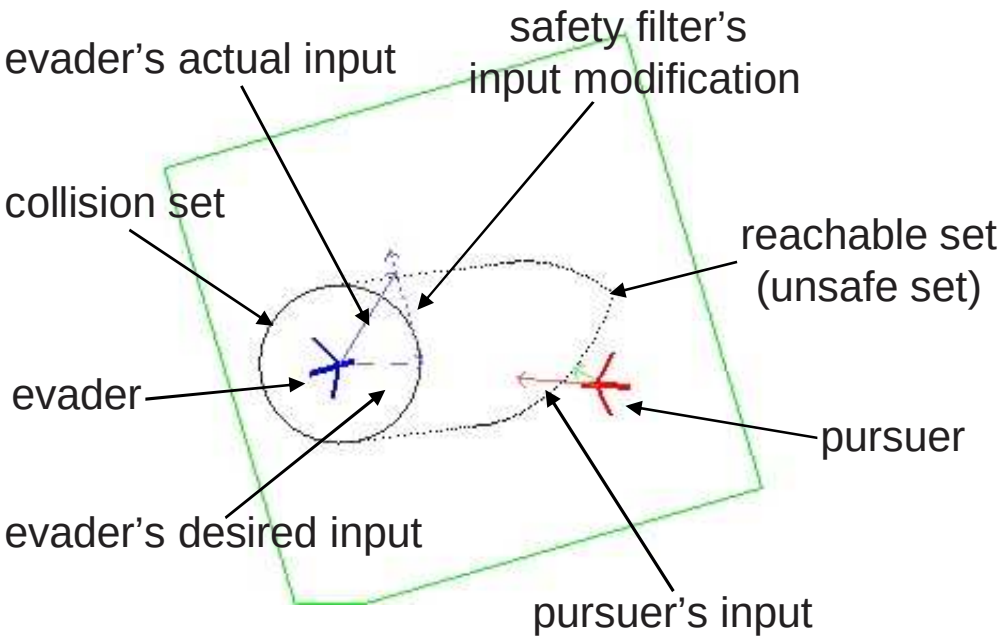
Reachable Sets: What and Why?

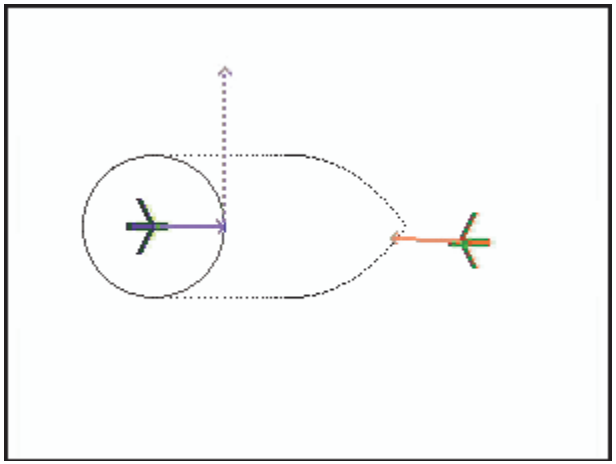
- One application: safety analysis
  - What states are doomed to become unsafe?
  - What states are safe given an appropriate control strategy?



Application: Softwalls for Aircraft Safety

- Use reachable sets to guarantee safety
- Basic Rules
  - Pursuer: turn to head toward evader
  - Evader: turn to head east
- Evader's input is filtered to guarantee that pursuer does not enter the reachable set

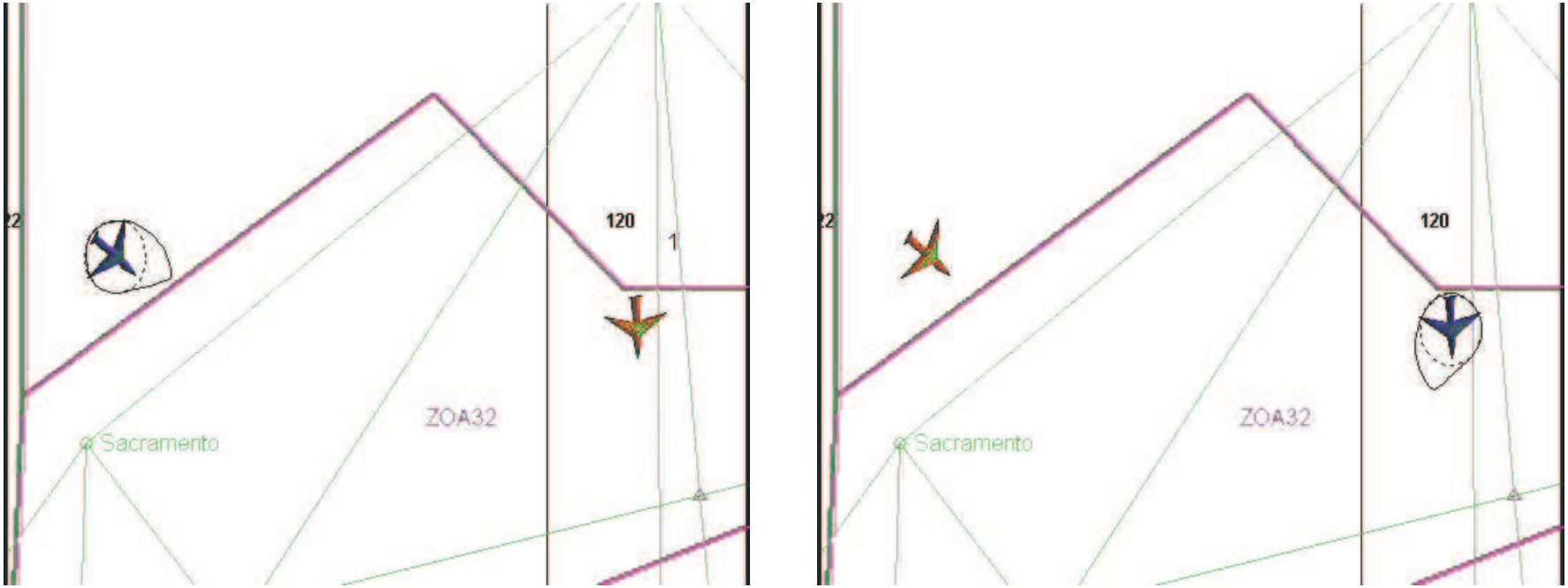




joint work with Edward Lee & Adam Cataldo

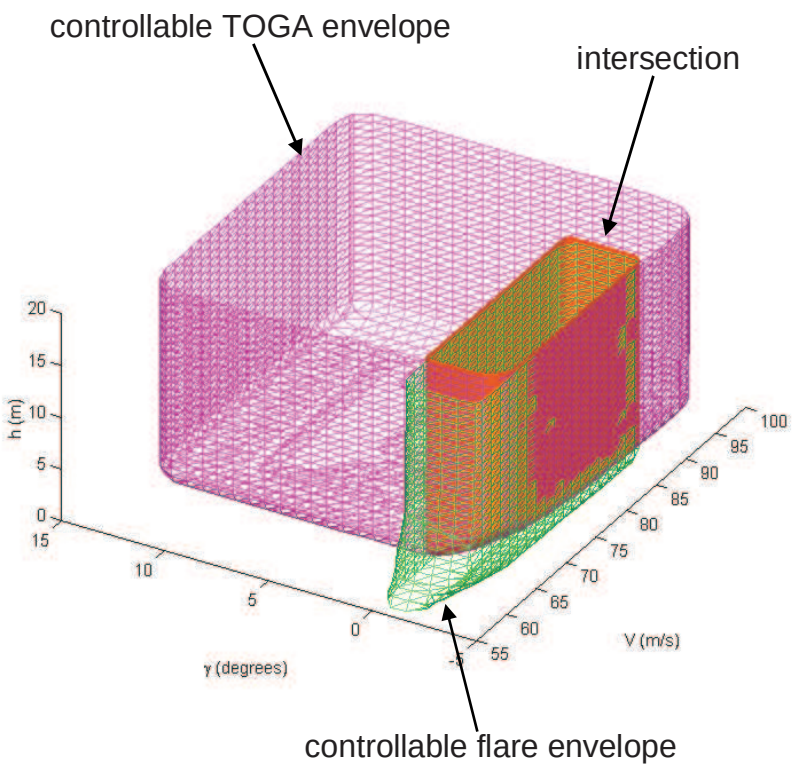
Application: Collision Alert for ATC

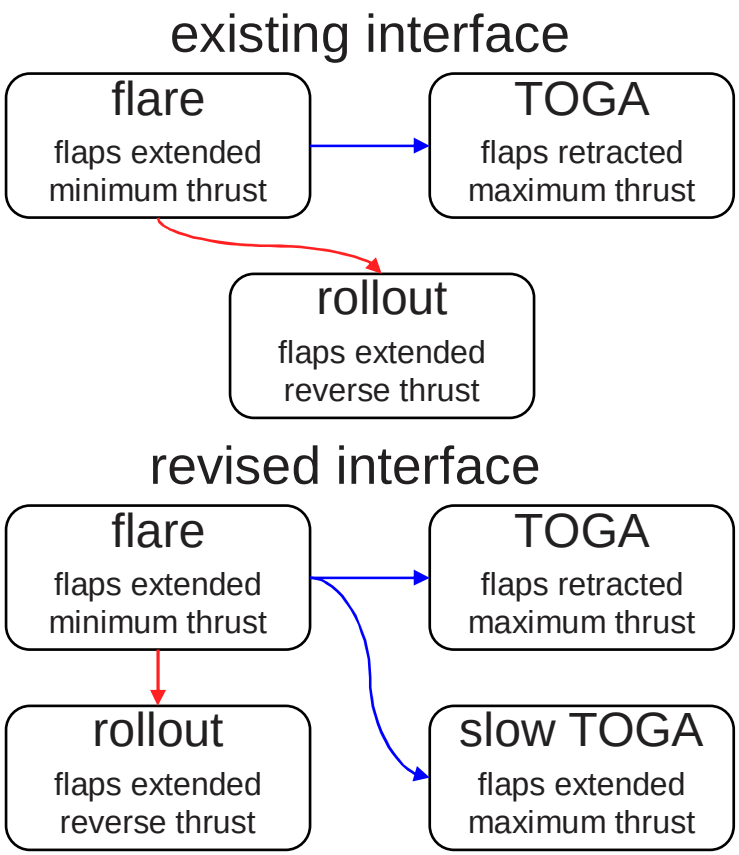
- Use reachable set to detect potential collisions and warn Air Traffic Control (ATC)
  - Find aircraft pairs in ETMS database whose flight plans intersect
  - Check whether either aircraft is in the other's collision region
  - If so, examine ETMS data to see if aircraft path is deviated
  - One hour sample in Oakland center's airspace—
    - 1590 pairs, 1555 no conflict, 25 detected conflicts, 2 false alerts



Application: Cockpit Display Analysis

- Controllable flight envelopes for landing and Take Off / Go Around (TOGA) maneuvers may not be the same
- Pilot's cockpit display may not contain sufficient information to distinguish whether TOGA can be initiated





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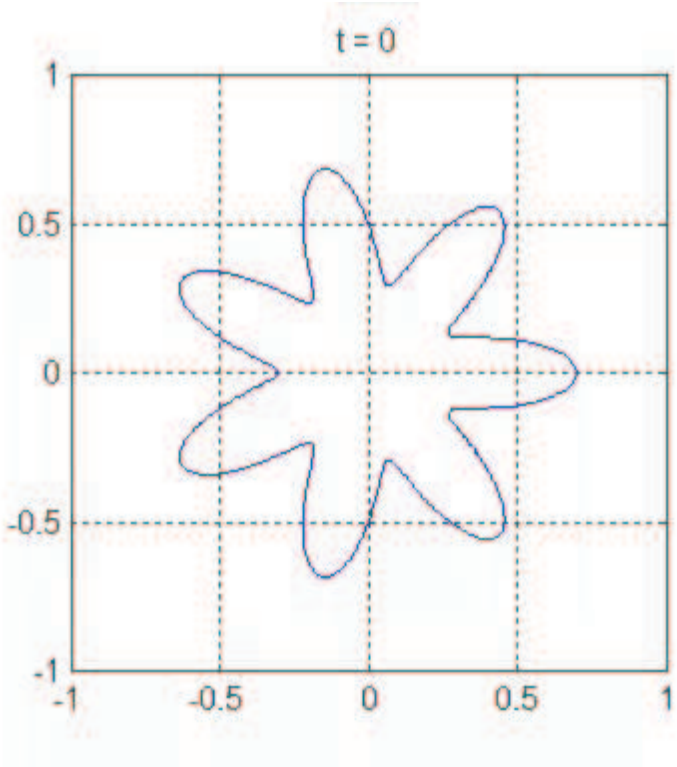
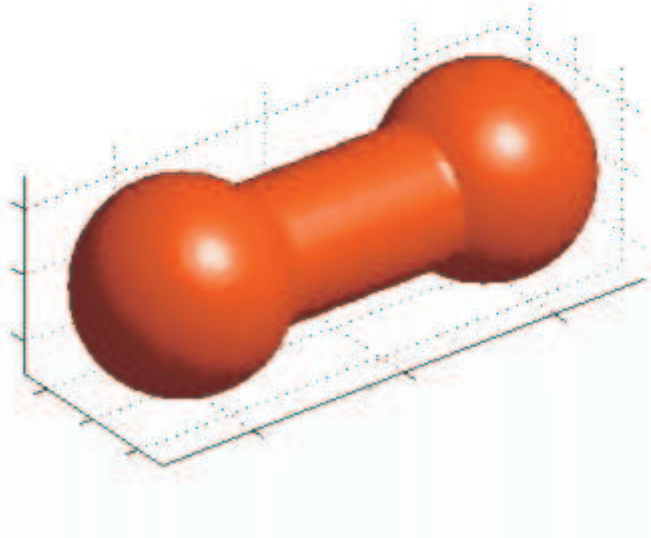
Dynamic Interfaces





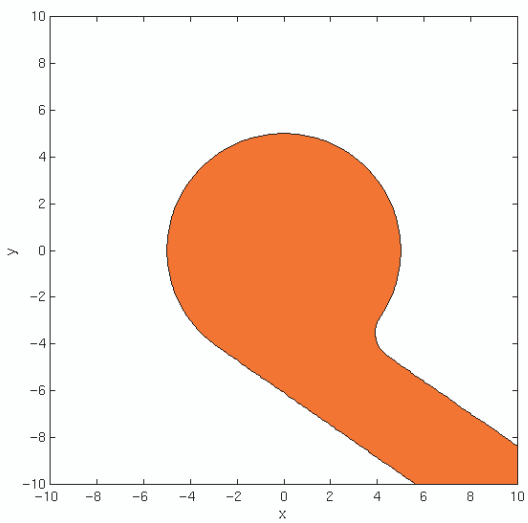
- How do you represent an evolving interface?
  - Size may grow or shrink
  - Regions may combine or pinch off
  - Interface is a curve in 2D, a surface in 3D
- One solution: dynamic implicit surfaces

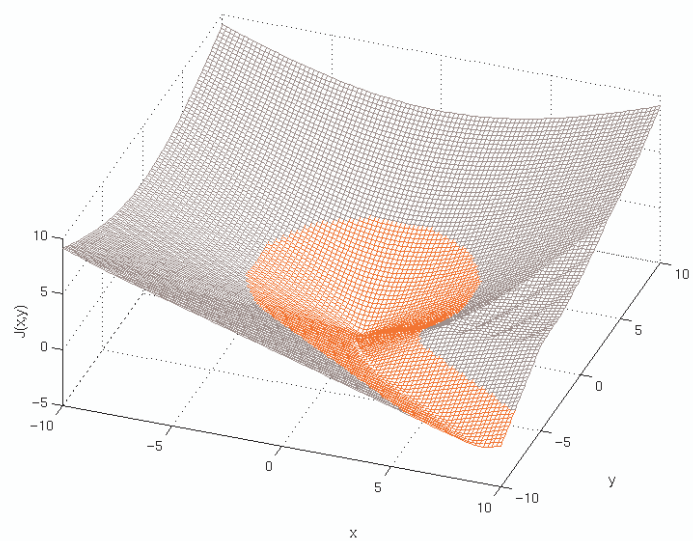
Implicit Surface Functions

- Surface $S(t)$ is defined implicitly by zero isosurface of a scalar function $\phi(x,t)$, with several benefits
  - Surfaces automatically merge and/or separate
  - Geometric quantities (normal, curvature) are easy to calculate
  - Most common surface motions become PDE in ϕ

$$\phi : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}$$

$$S(t) = \{x \in \mathbb{R}^n \mid \phi(x,t) = 0\}$$

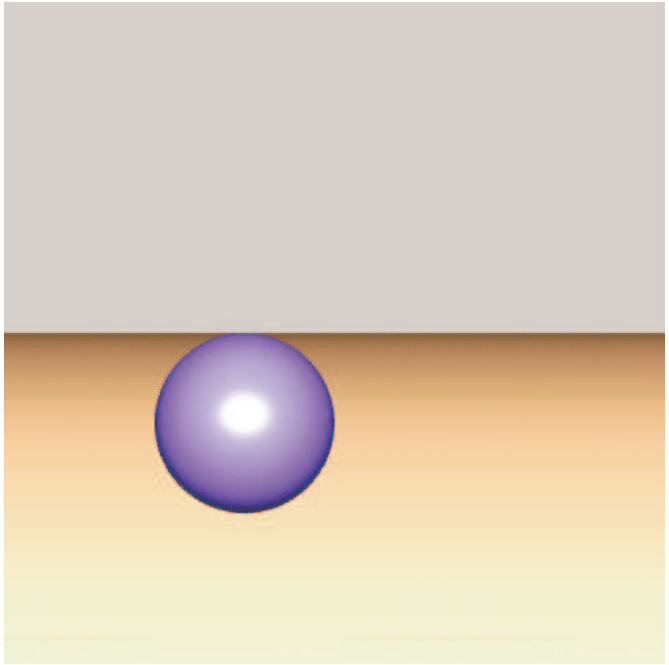




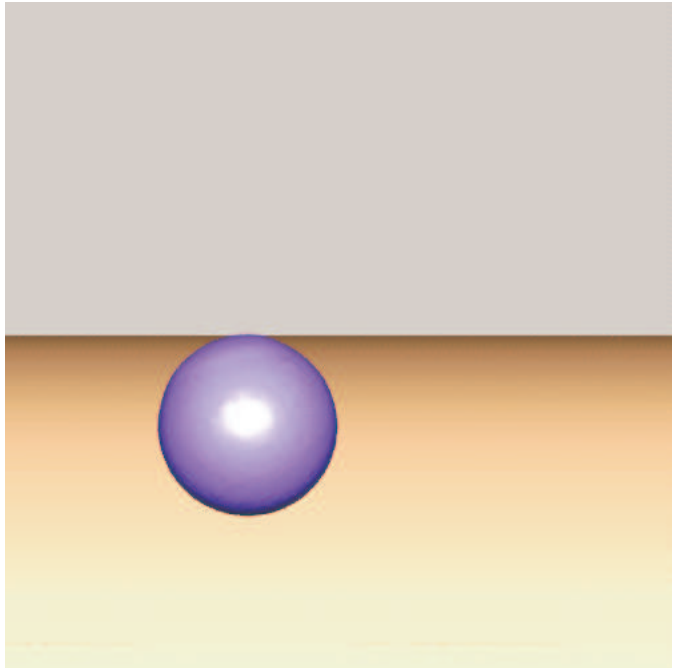
Level Set Methods

- Challenge: how can we conserve volume despite small features and numerical error?

Level Set Only



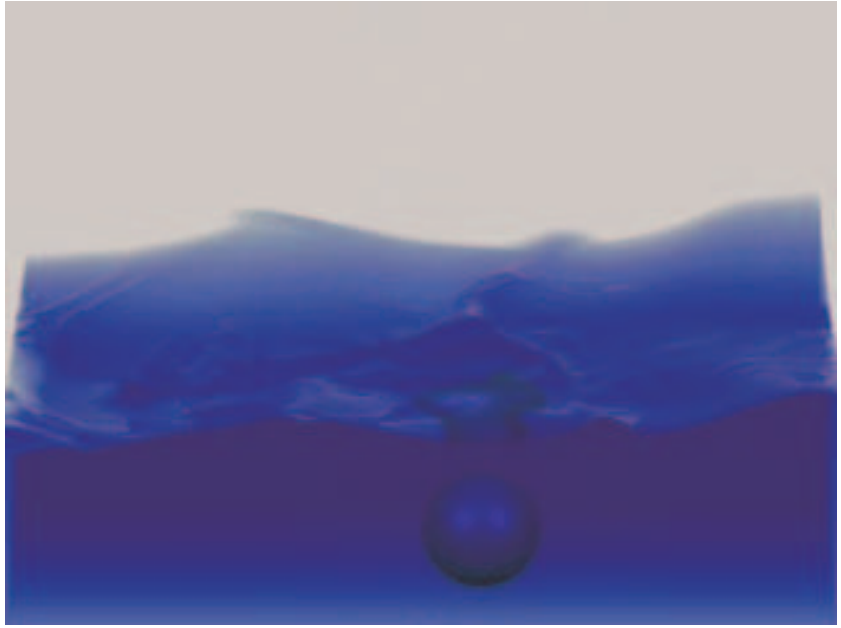
Particle Level Set



Rendering by Sou Cheng Choi

Application: Animating Fluids

- State of the art evolving interface
  - Merging and separating surfaces
  - Smooth simulation and rendering of fluid and container
  - Plausible water motion
- Requires fluid simulation as well (not my research)

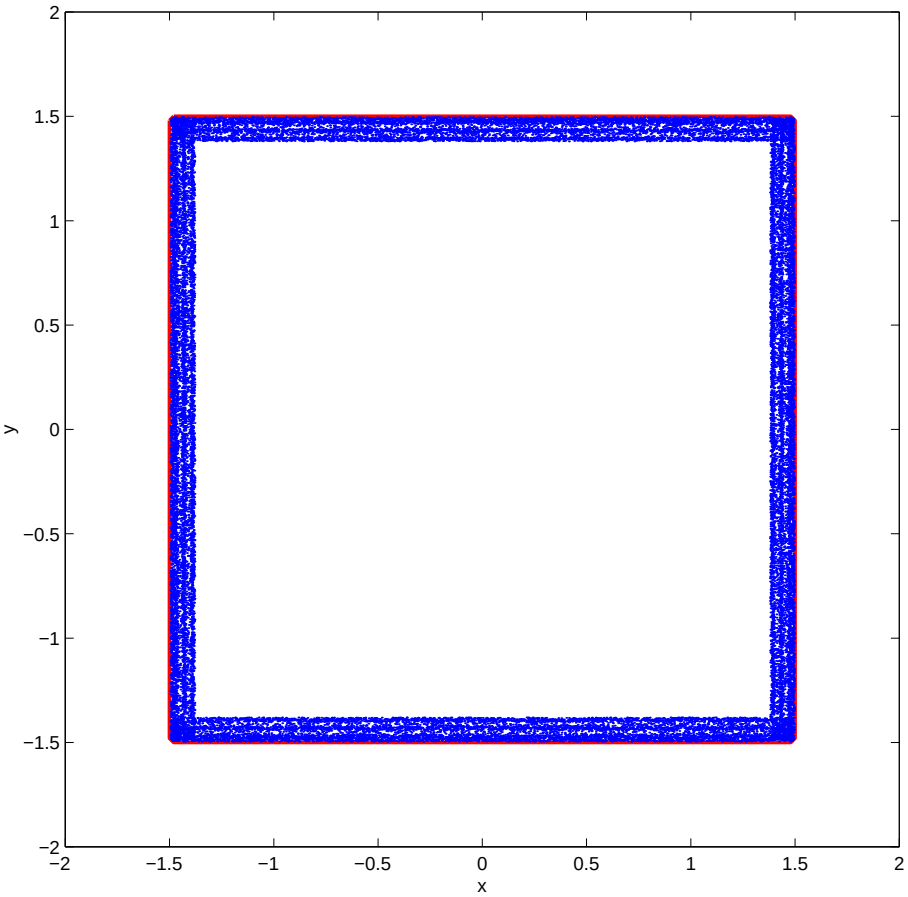


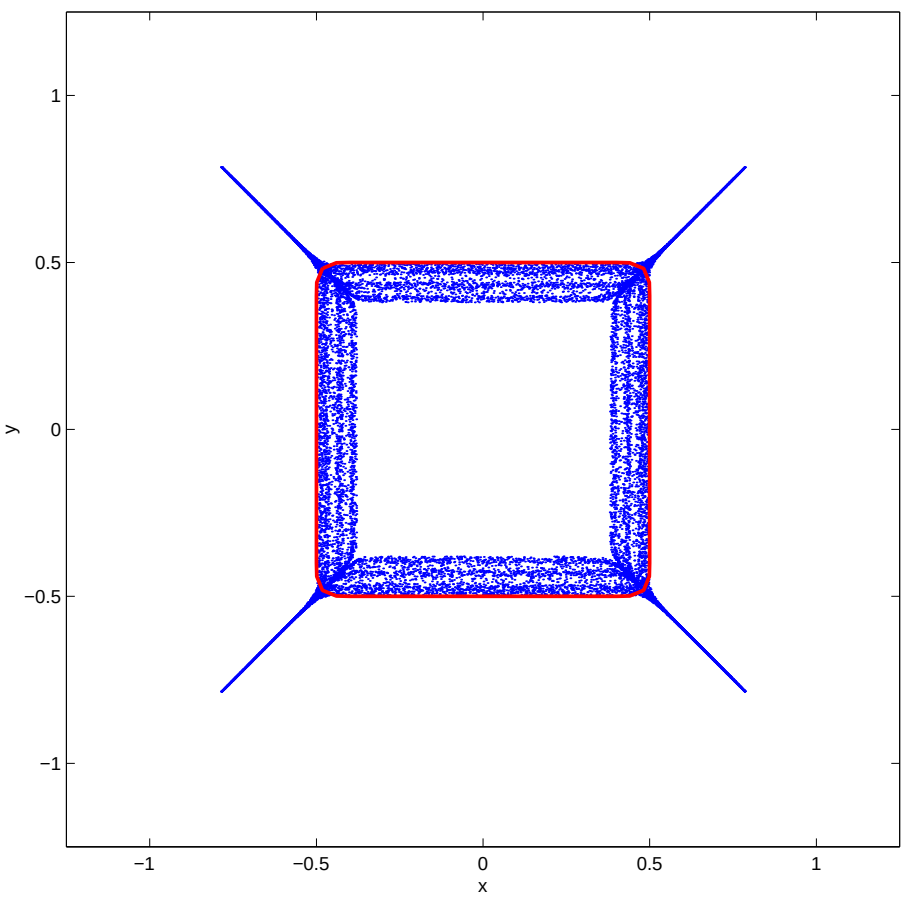
Foster & Fedkiw (2001)

Enright, Marschner & Fedkiw (2002)

Not Finished Yet

- Reports of dubious repeatability.
- What about shocks? Particle methods fail.





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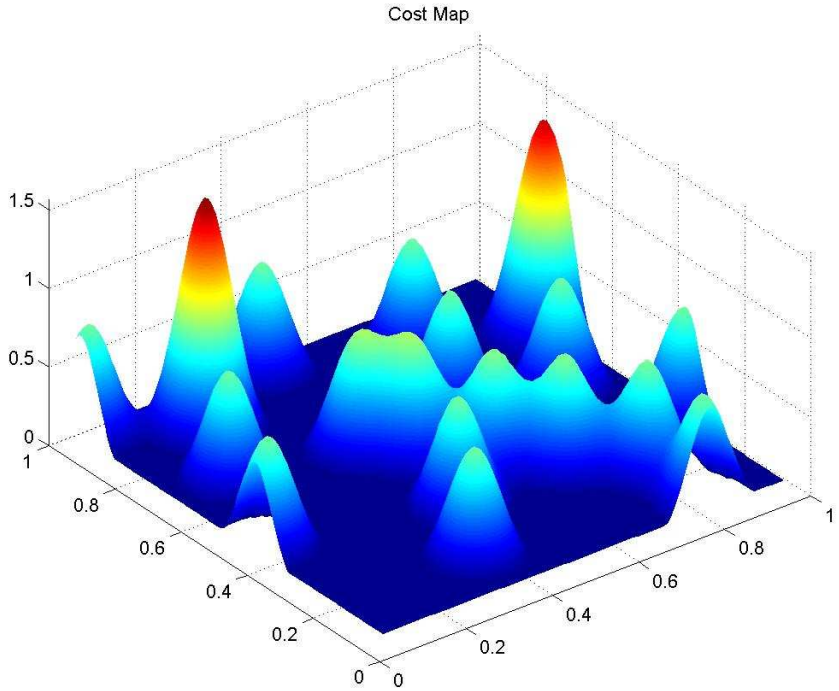
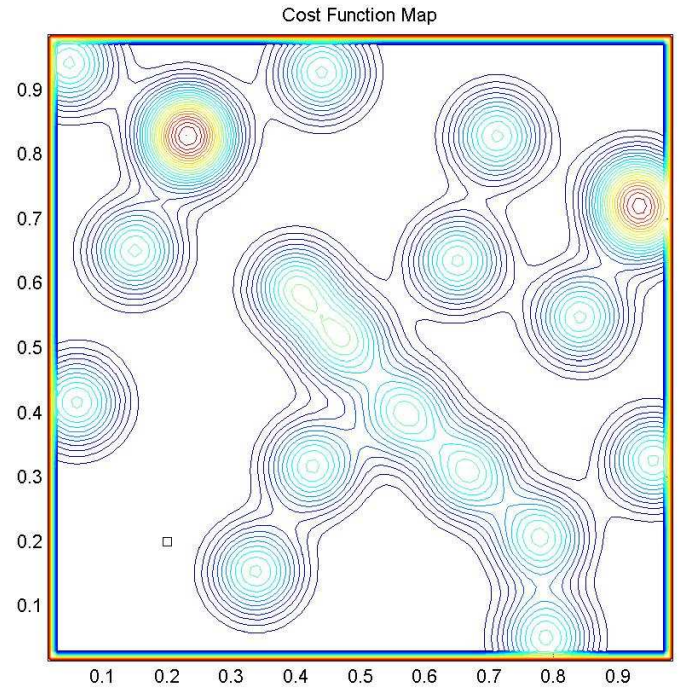
Basic Path Planning

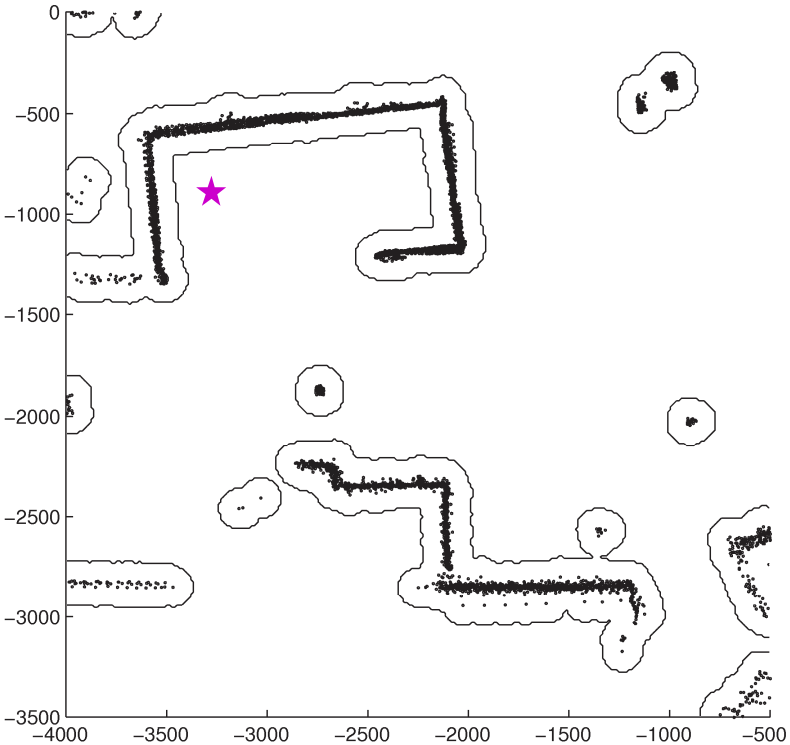
- Find the optimal path $p(s)$ to a target (or from a source)
- Inputs
  - Cost to pass through each state in the state space
  - Set of targets or sources (provides boundary conditions)



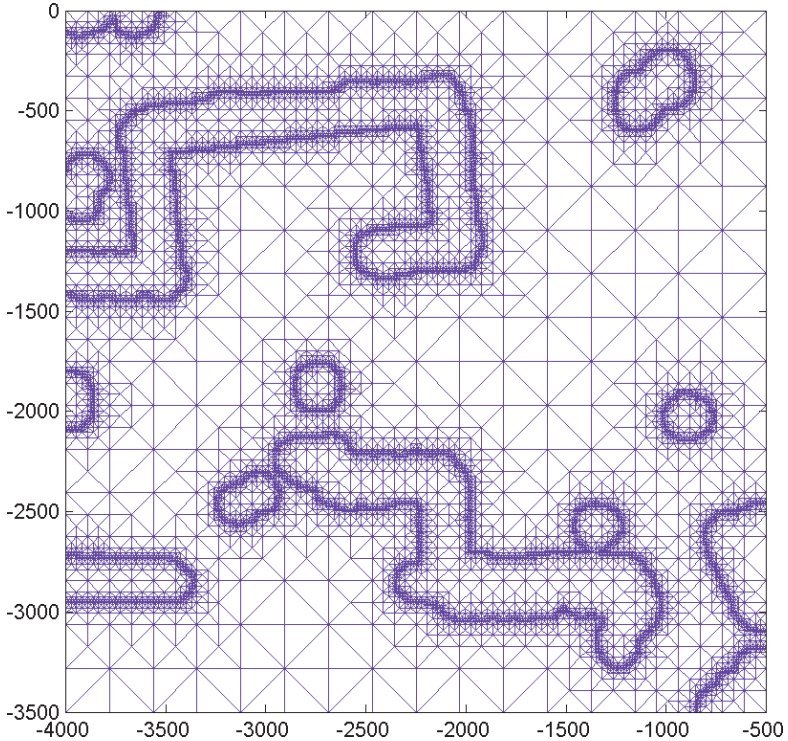


Robot Path Planning

- Find shortest path to objective while avoiding obstacles
  - Obstacle maps from laser scanner
  - Configuration space accounts for robot shape
  - Cost function essentially binary



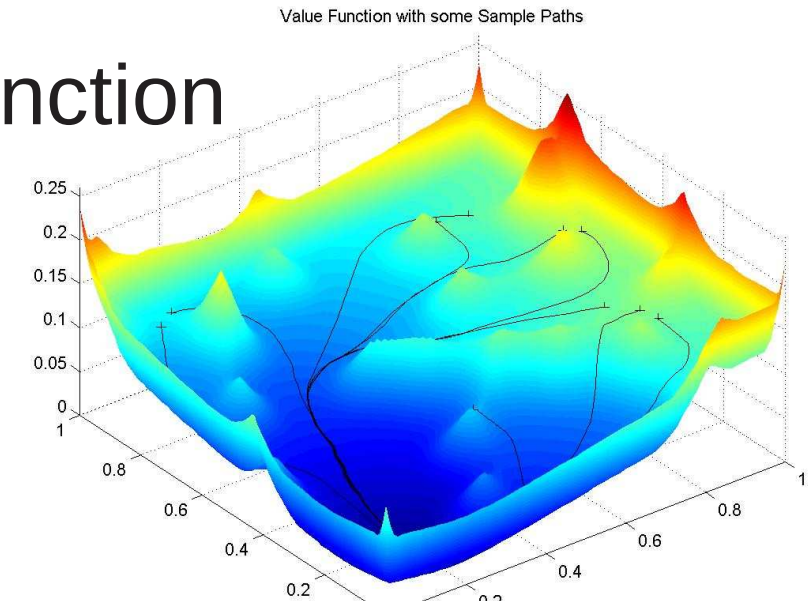
typical laser scan with configuration space obstacles

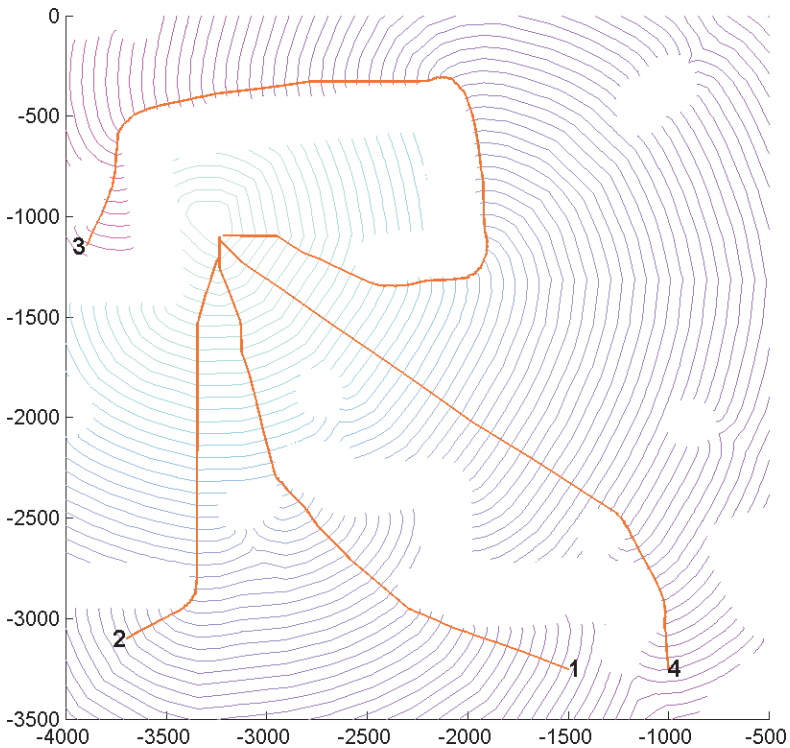


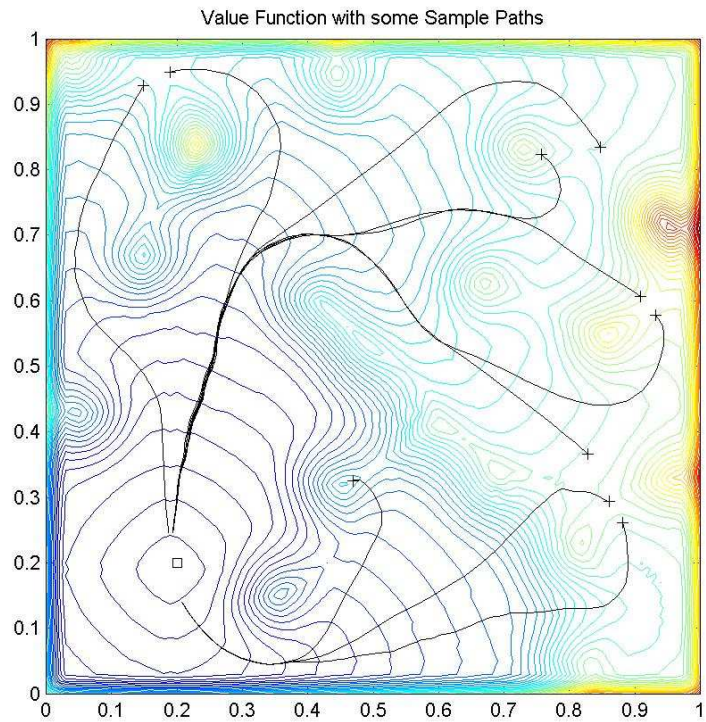
adaptive grid

Value Function

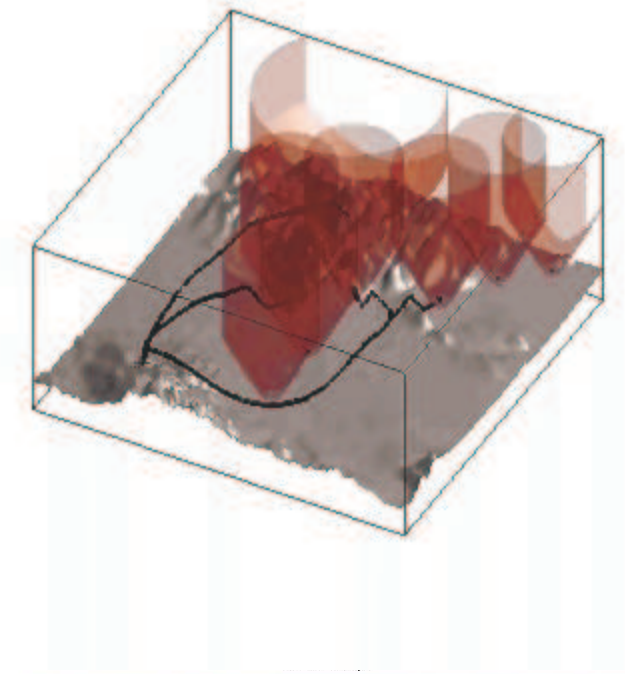
- Specifies cost of optimal path to target from any point
- Steepest descent finds optimal path

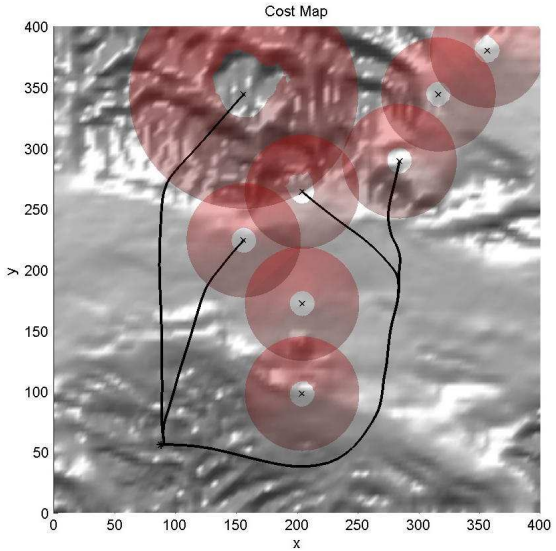


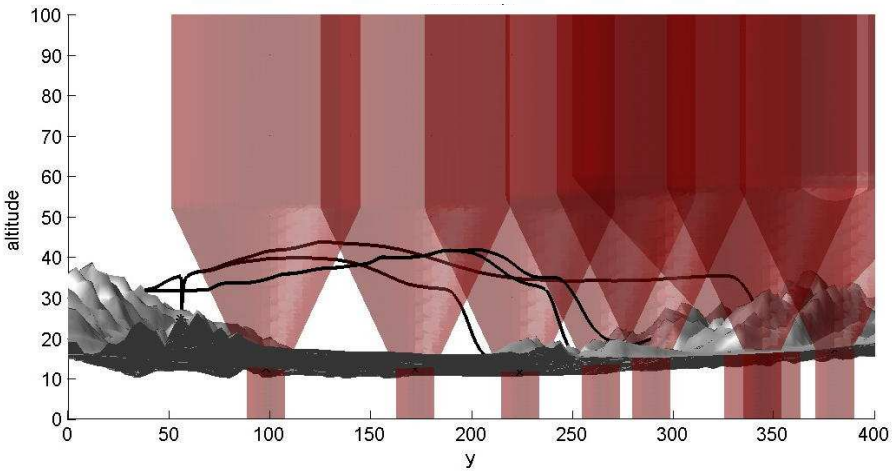


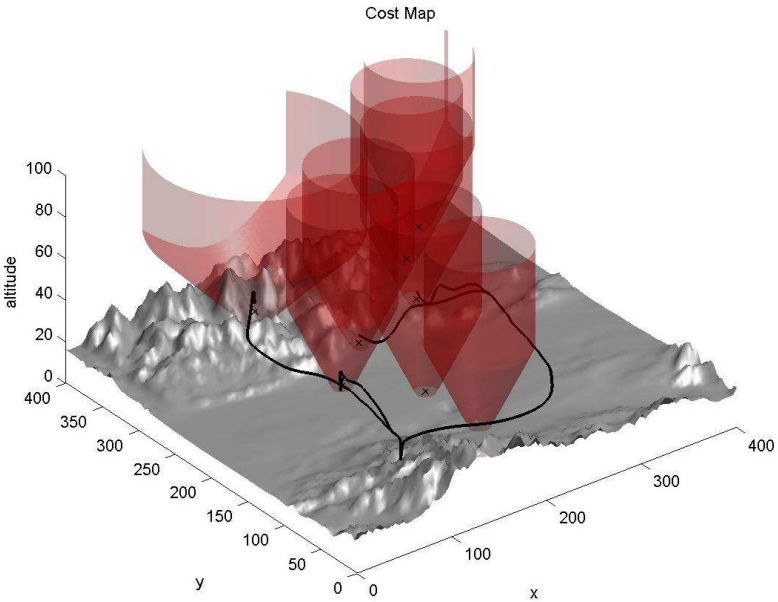


Demanding Example? No!





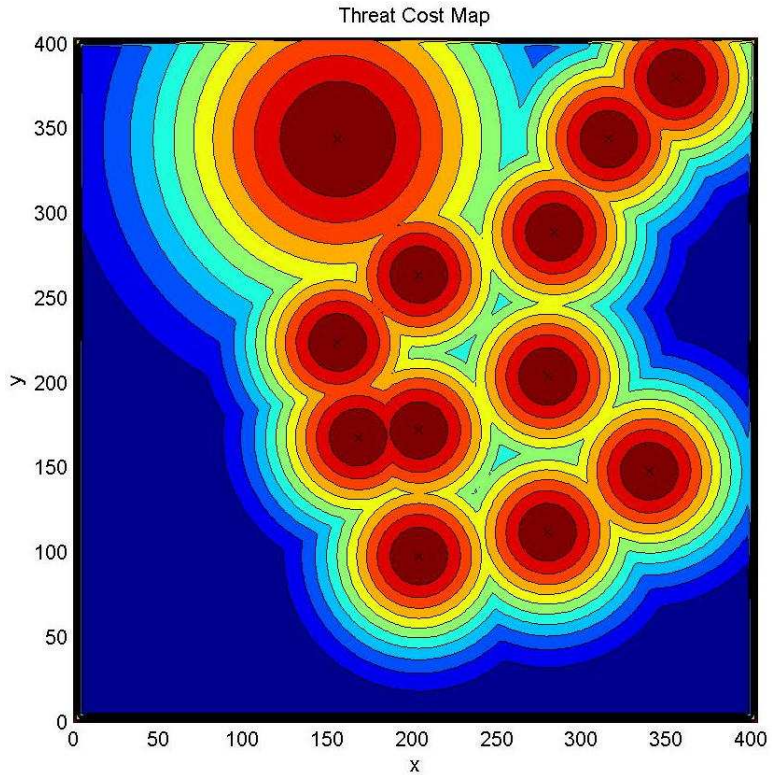




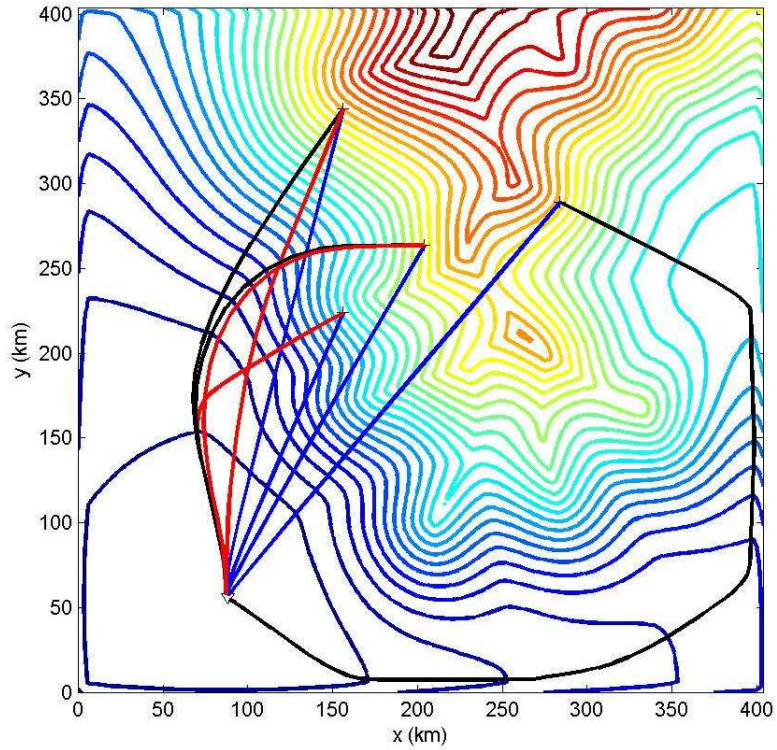
C Computer

Constrained Example

- Plan path to selected sites
  - Threat cost function is maximum of individual threats
- For each target, plan 3 paths
  - minimum threat, minimum fuel, minimum threat (with fuel ≤ 300)



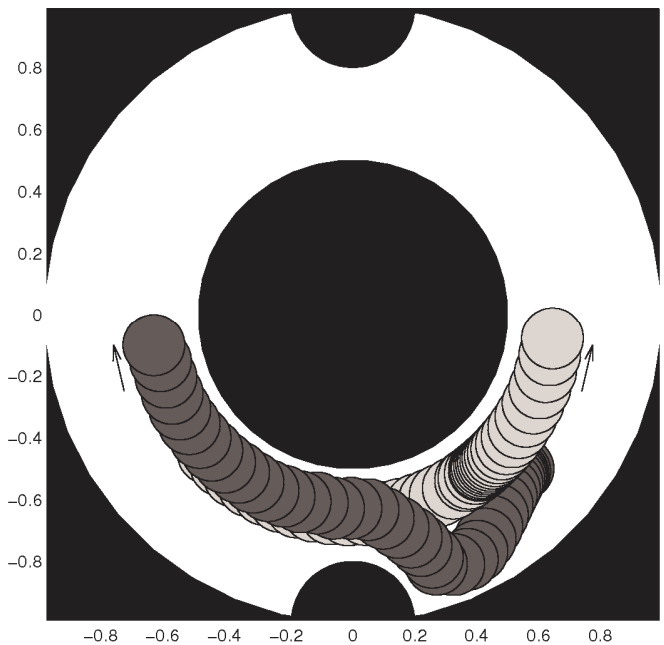
threat cost

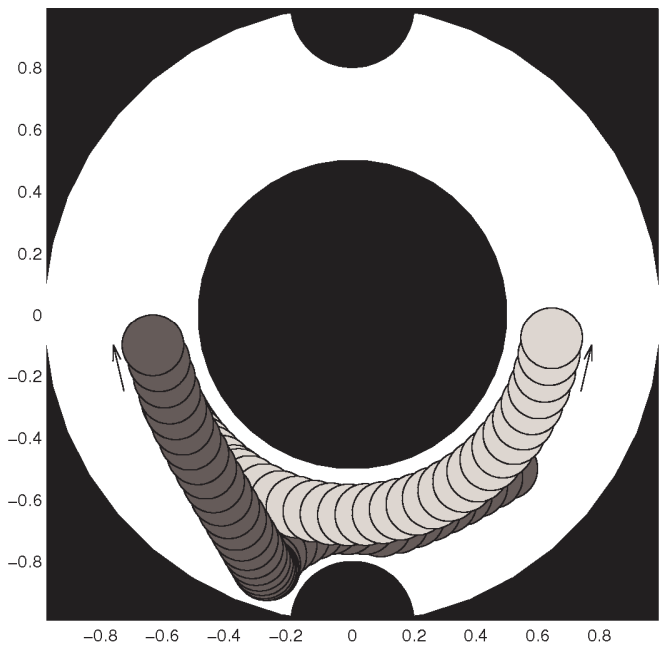


Paths (on value function)

Mixtures of Norms

- May even be situations where action norm bounds are mixed
  - Dark shaded robot starts on right, may move any direction in 2D
  - Light shaded robot starts on left, constrained to 1D circular path
  - Cost encodes black obstacles and collision states
  - 2D robot action constrained in $||\cdot||_2$ and combined action in $||\cdot||_\infty$





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Evolving Reachable Sets

- Modified Hamilton-Jacobi partial differential equation

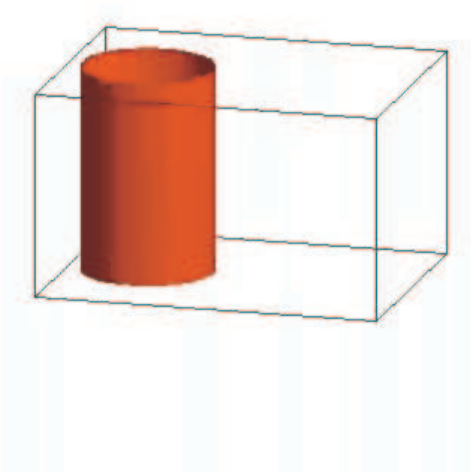
$$D_t\phi(x,t) + \min\left[0, H(x, D_x\phi(x,t))\right] = 0$$

$$\text{with Hamiltonian}: \; H(x,p) = \max_{a\in\mathcal{A}} \min_{b\in\mathcal{B}} f(x,a,b)\cdot p$$

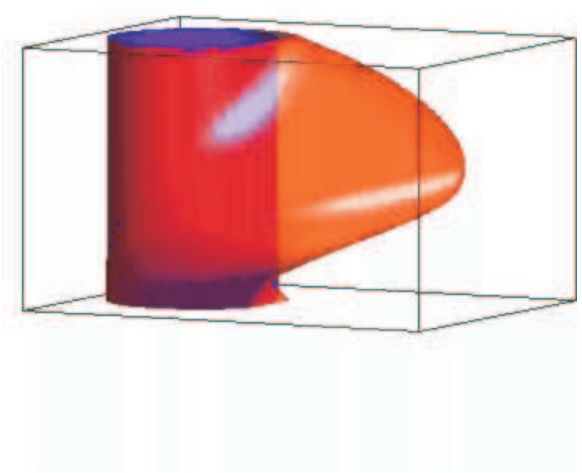
$$\text{and terminal conditions}: \; \phi(x,0) = h(x)$$

$$\text{where} \quad G(0) = \{x \in \mathbb{R}^n \mid h(x) \leq 0\}$$

$$\text{and} \quad \dot{x} = f(x,a,b)$$

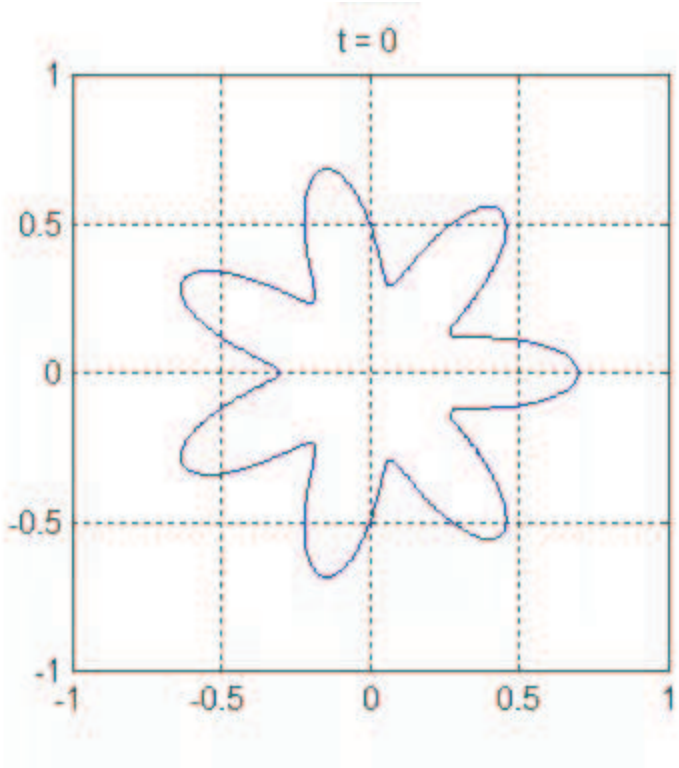


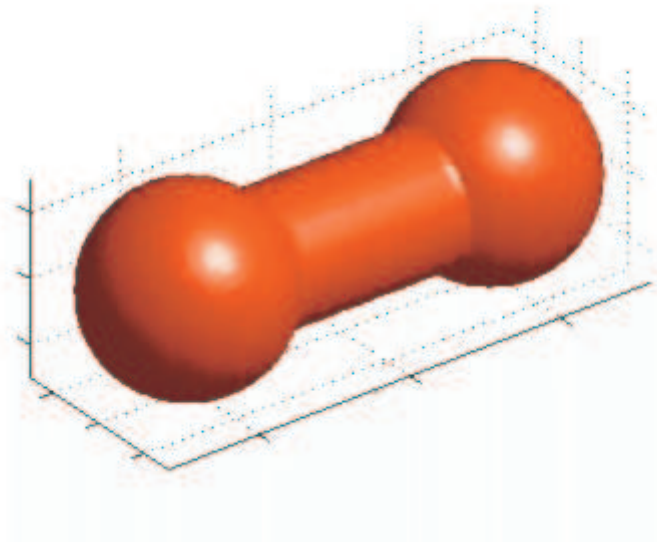
growth of reachable set



final reachable set

Dynamic Implicit Surfaces





$$D_t\phi(x,t) + a(x,t)\|\nabla\phi(x,t)\| + v(x,t)\cdot\nabla\phi(x,t) = 0$$

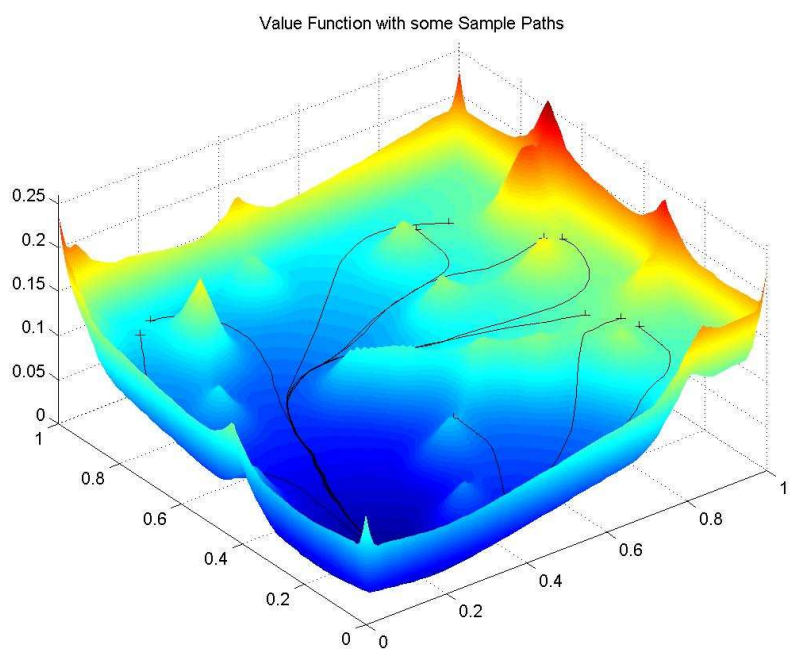
$$D_t\phi(x,t) - \kappa(x,t)\|\nabla\phi(x,t)\| = 0$$

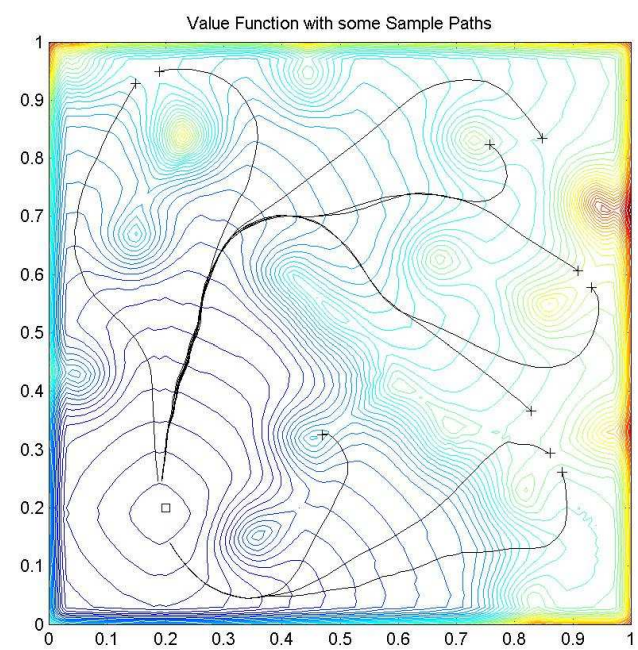
Path Generation

- Value function $V(x)$ is solution of Eikonal equation
  - Solution may not be differentiable
- Optimal path $p(s)$ is found by gradient descent
  - Value function $V(x)$ has no local minima, so paths will always terminate at a target

$$\|\nabla V(x)\| = c(x)$$

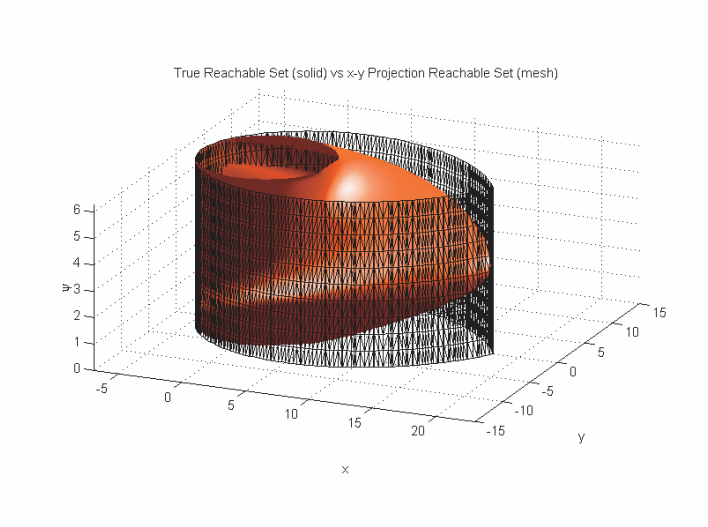
$$\frac{dp}{ds} = \frac{\nabla V(x)}{\|\nabla V(x)\|}$$

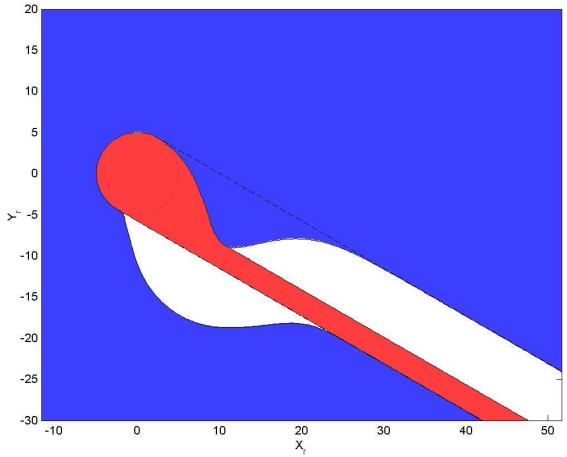


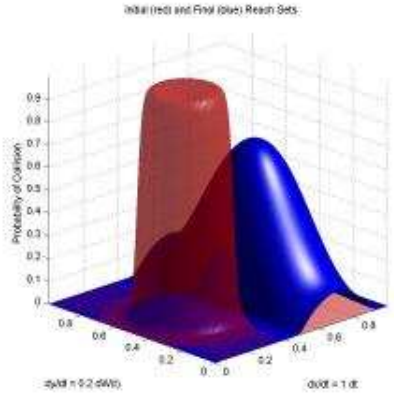


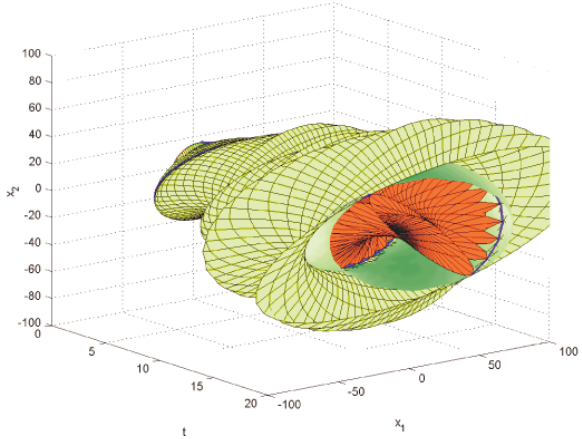
Not Discussed

- Mathematical Finance & Ecology
- Image Processing
- Embedded and real-time platforms
- Methods for higher dimensions
- Hybrid and Stochastic Reach sets









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