Personal Research Areas

Ian Mitchell
Department of Computer Science
The University of British Columbia

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Outline

• Reach sets for control verification
  – Filtering pilot commands for safety

• Dynamic surfaces
  – Particle level set
  – Free surface fluid simulation

• Robotic path planning
  – Shortest continuous path, subject to constraints
  – Optimality for multiple vehicle and/or various norms

• Hamilton-Jacobi PDEs
  – The common link

• Just for fun: none of this material is from the course
  – It is graduate level and/or research material
Reachable Sets: What and Why?

- One application: safety analysis
  - What states are doomed to become unsafe?
  - What states are safe given an appropriate control strategy?
Application: Softwalls for Aircraft Safety

- Use reachable sets to guarantee safety
- Basic Rules
  - Pursuer: turn to head toward evader
  - Evader: turn to head east
- Evader’s input is filtered to guarantee that pursuer does not enter the reachable set

joint work with Edward Lee & Adam Cataldo
Application: Collision Alert for ATC

- Use reachable set to detect potential collisions and warn Air Traffic Control (ATC)
  - Find aircraft pairs in ETMS database whose flight plans intersect
  - Check whether either aircraft is in the other’s collision region
  - If so, examine ETMS data to see if aircraft path is deviated
  - One hour sample in Oakland center’s airspace—
    - 1590 pairs, 1555 no conflict, 25 detected conflicts, 2 false alerts
Application: Cockpit Display Analysis

- Controllable flight envelopes for landing and Take Off / Go Around (TOGA) maneuvers may not be the same
- Pilot’s cockpit display may not contain sufficient information to distinguish whether TOGA can be initiated
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Dynamic Interfaces

• How do you represent an evolving interface?
  – Size may grow or shrink
  – Regions may combine or pinch off
  – Interface is a curve in 2D, a surface in 3D

• One solution: dynamic implicit surfaces
Implicit Surface Functions

- Surface $S(t)$ is defined implicitly by zero isosurface of a scalar function $\phi(x,t)$, with several benefits
  - Surfaces automatically merge and/or separate
  - Geometric quantities (normal, curvature) are easy to calculate
  - Most common surface motions become PDE in $\phi$

$$\phi : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}$$

$$S(t) = \{ x \in \mathbb{R}^n \mid \phi(x, t) = 0 \}$$
Level Set Methods

- Challenge: how can we conserve volume despite small features and numerical error?
Application: Animating Fluids

- State of the art evolving interface
  - Merging and separating surfaces
  - Smooth simulation and rendering of fluid and container
  - Plausible water motion
- Requires fluid simulation as well (not my research)

Not Finished Yet

- Reports of dubious repeatability.
- What about shocks? Particle methods fail.
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Basic Path Planning

- Find the optimal path $p(s)$ to a target (or from a source)
- Inputs
  - Cost to pass through each state in the state space
  - Set of targets or sources (provides boundary conditions)
Robot Path Planning

- Find shortest path to objective while avoiding obstacles
  - Obstacle maps from laser scanner
  - Configuration space accounts for robot shape
  - Cost function essentially binary

*typical laser scan with configuration space obstacles*

*adaptive grid*
Value Function

- Specifies cost of optimal path to target from any point
- Steepest descent finds optimal path
Demanding Example? No!
Constrained Example

- Plan path to selected sites
  - Threat cost function is maximum of individual threats
- For each target, plan 3 paths
  - minimum threat, minimum fuel, minimum threat (with fuel ≤ 300)

threat cost

Paths (on value function)
Mixtures of Norms

- May even be situations where action norm bounds are mixed
  - Dark shaded robot starts on right, may move any direction in 2D
  - Light shaded robot starts on left, constrained to 1D circular path
  - Cost encodes black obstacles and collision states
  - 2D robot action constrained in $\|\cdot\|_2$ and combined action in $\|\cdot\|_\infty$
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Evolving Reachable Sets

- Modified Hamilton-Jacobi partial differential equation

\[ D_t \phi(x, t) + \min [0, H(x, D_x \phi(x, t))] = 0 \]

with Hamiltonian: \( H(x, p) = \max_{a \in A} \min_{b \in B} f(x, a, b) \cdot p \)

and terminal conditions: \( \phi(x, 0) = h(x) \)

where \( G(0) = \{ x \in \mathbb{R}^n \mid h(x) \leq 0 \} \)

and \( \dot{x} = f(x, a, b) \)
Dynamic Implicit Surfaces

\[ D_t \phi(x, t) \]
\[ + a(x, t) \| \nabla \phi(x, t) \| \]
\[ + v(x, t) \cdot \nabla \phi(x, t) = 0 \]

\[ D_t \phi(x, t) \]
\[ - \kappa(x, t) \| \nabla \phi(x, t) \| = 0 \]
Path Generation

- Value function $V(x)$ is solution of Eikonal equation
  - Solution may not be differentiable
- Optimal path $p(s)$ is found by gradient descent
  - Value function $V(x)$ has no local minima, so paths will always terminate at a target

\[ \| \nabla V(x) \| = c(x) \]

\[ \frac{dp}{ds} = \frac{\nabla V(x)}{\| \nabla V(x) \|} \]
Not Discussed

- Mathematical Finance & Ecology
- Image Processing
- Embedded and real-time platforms
- Methods for higher dimensions
- Hybrid and Stochastic Reach sets

mitchell@cs.ubc.ca
http://www.cs.ubc.ca/~mitchell