Module 4: Propositional Logic Proofs
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- Pre-class quiz #5 is due Sunday February 1st at 19:00
  - Assigned reading for the quiz:
    - Epp, 4th edition: 3.1, 3.3
    - Epp, 3rd edition: 2.1, 2.3
    - Rosen, 6th edition: 1.3, 1.4
    - Rosen, 7th edition: 1.4, 1.5
- Assignment #2 is due Thursday February 5th at 17:00.
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- Pre-class quiz #6 is tentatively due Tuesday February 10th at 19:00
  - Assigned reading for the quiz:
    - Epp, 4th edition: 3.2, 3.4
    - Epp, 3rd edition: 2.2, 2.4
    - Rosen, 6th edition: 1.3, 1.4
    - Rosen, 7th edition: 1.4, 1.5
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By the start of this class you should be able to:

- Use truth tables to establish or refute the validity of a rule of inference.
- Given a rule of inference and propositional logic statements that correspond to the rule's premises, apply the rule to infer a new statement implied by the original statements.
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Quiz 4 feedback:
- Overall well done
- We will talk about two questions whose averages were slightly lower than the rest.
- We will discuss the open-ended question as well.
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Use the table below to determine whether this proposed rule of inference is valid.

\[
\begin{array}{c}
p \land \sim p \\
\hline \\
q
\end{array}
\]

If the rule is invalid, select any one line of the truth table which proves that the rule is invalid.

a) 1  

b) 2  

c) 3  

d) 4  

e) The rule is valid
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Consider the following rule of inference:

\[ a \rightarrow b \]
\[ b \rightarrow c \]
\[ \therefore a \rightarrow c \]

What do we get by applying this rule to the statements:

\[ p \rightarrow (q \lor r) \]
\[ q \rightarrow s \]

a) \( p \rightarrow q \)  
 b) \( p \rightarrow r \)  
 c) \( p \rightarrow s \)  
 d) \( p \rightarrow (q \lor r) \)  
 e) Some other statement.  
 f) None of these because the rule doesn't apply.
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- CPSC 121: the BIG questions:
  - How can we convince ourselves that an algorithm does what it's supposed to do?
    - We need to prove that it works.
    - We have done a few proofs in the last week or so.
    - Now we will learn
      - How to decide if a proof is valid in a formal setting.
      - How to write proofs in English.
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- By the end of this module, you should be able to
  - Determine whether or not a propositional logic proof is valid, and explain why it is valid or invalid.
  - Explore the consequences of a set of propositional logic statements by application of equivalence and inference rules, especially in order to massage statements into a desired form.
  - Devise and attempt multiple different, appropriate strategies for proving a propositional logic statement follows from a list or premises.
What is a proof?

A rigorous formal argument that demonstrates the truth of a proposition, given the truth of the proof’s premises.

In other words:
- A proof is used to convince other people (or yourself) of the truth of a conditional proposition.
- Every step must be well justified.

Writing a proof is a bit like writing a function:
- you do it step by step, and
- make sure that you understand how each step relates to the previous steps.
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- Things we might prove
  - We can build a combinational circuit matching any truth table.
  - We can build any digital logic circuit using only 2-input NOR gates.
  - The maximum number of swaps we need to order $n$ students is $n(n-1)/2$.
  - No general algorithm exists to sort $n$ values using fewer than $n \log_2 n$ comparisons.
  - There are problems that no algorithm can solve.
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Suppose that you proved this:

Premise 1

...  

Premise n

\[ \therefore \text{Conclusion} \]

Does it mean:

a) Premises 1 to n are true  
   b) Conclusion is true  
   c) Premises 1 to n can be true  
   d) Conclusion can be true  
   e) None of the above.
A propositional logic proof is a sequence of propositions, where each proposition is one of:

- A premise
- The result of applying a logical equivalence or a rule of inference to one or more earlier propositions.

and whose last proposition is the conclusion.

These are good starting point, because they are simpler than the more free-form proofs we will discuss later.

- Only a limited number of choices at each step.
Onnagata problem from pre-class quiz #4

Critique the following argument, drawn from an article by Julian Baggini on logical fallacies.

Premise 1: If women are too close to femininity to portray women then men must be too close to masculinity to play men, and vice versa.

Premise 2: And yet, if the onnagata are correct, women are too close to femininity to portray women and yet men are not too close to masculinity to play men.

Conclusion: Therefore, the onnagata are incorrect, and women are not too close to femininity to portray women.

Note: onnagata are male actors portraying female characters in kabuki theatre.
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- Onnagata: which definitions should we use?
  a) \( w = \text{women}, \ m = \text{men}, \ f = \text{femininity}, \ m = \text{masculinity}, \ o = \text{onnagata}, \ c = \text{correct} \)
  b) \( w = \text{women are too close to femininity}, \ m = \text{men are too close to masculinity}, \ pw = \text{women portray women}, \ pm = \text{men portray men}, \ o = \text{onnagata are correct} \)
  c) \( w = \text{women are too close to femininity to portray women}, \ m = \text{men are too close to masculinity to portray men}, \ o = \text{onnagata are correct} \)
  d) None of these, but another set of definitions works well.
  e) None of these, and this problem cannot be modeled well with propositional logic.
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- Onnagata: do the two premises contradict each other (that is, is $p_1 \land p_2 \equiv F$)?
  
  a) Yes  
  b) No  
  c) Not enough information to tell

- What can we prove?
  
  - We can prove that the Onnagata are wrong.  
  - We can **not** prove that women are not too close to femininity to portray women.

  - What other scenario is consistent with the premises?
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- Proof strategies
  - Look at the information you have
    - Is there irrelevant information you can ignore?
    - Is there critical information you should focus on?
  - Work backwards from the end
    - Especially if you have made some progress but are missing a step or two.
  - Don't be afraid of inferring new propositions, even if you are not quite sure whether or not they will help you get to the conclusion you want.
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- Proof strategies (continued):
  - If you are not sure of the conclusion, alternate between
    - trying to find an example that shows the statement is false, using the place where your proof failed to help you design the counterexample.
    - trying to prove it, using your failed counterexample to help you write the proof.
Example: prove that the following argument is valid:

\[ p \]
\[ p \rightarrow r \]
\[ p \rightarrow \sim s \]
\[ p \rightarrow (q \lor \sim r) \]
\[ \sim q \lor s \]
\[ \therefore s \]
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- Why can we not just use truth tables to prove propositional logic theorems?
  a) No reason; truth tables are enough.
  b) Truth tables scale poorly to large problems.
  c) Rules of inference and equivalence rules can prove theorems that cannot be proven with truth tables.
  d) Truth tables require insight to use, while rules of inference can be applied mechanically.
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Why not use logical equivalences to prove that the conclusions follow from the premises?

a) No reason; logical equivalences are enough.

b) Logical equivalences scale poorly to large problems.

c) Rules of inference and truth tables can prove theorems that cannot be proven with logical equivalences.

d) Logical equivalences require insight to use, while rules of inference can be applied mechanically.
One last question:

Consider the following:

Patrice is rich

If Patrice is rich then he will pay your tuition

\[ \therefore \text{Patrice will pay your tuition.} \]

Is this argument valid?

Should you pay your tuition, or should you assume that Patrice will pay it for you? Why?
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- Further exercises
  - Prove that the following argument is valid:
    
    \[
    \begin{align*}
    & p \rightarrow q \\
    & q \rightarrow (r \land s) \\
    & \sim r \lor (\sim t \lor u) \\
    & p \land t \\
    \therefore & u
    \end{align*}
    \]
  - Given the following, what is everything you can prove?
    
    \[
    \begin{align*}
    & p \rightarrow q \\
    & p \lor \sim q \lor r \\
    & (r \land \sim p) \lor s \lor \sim p \\
    & \sim r
    \end{align*}
    \]
Further exercises

Hercule Poirot has been asked by Lord Maabo to find out who closed the lid of his piano after dumping the cat inside. Poirot interrogates two of the servants, Pearrh and Scarfin. One and only one of them put the cat in the piano. Plus, one always lies and one never lies.

- Scarfin: I did not put the cat in the piano. Tgahaa gave me less than $60 to help her study.
- Pearrh: Scarfin did it. Tgahaa paid him $50 to help her study.

Who put the cat in the piano?