CPSC 426: Computer Animation
Quiz 1

March 3, 2014

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name: Solutions

Student Number: ____________________

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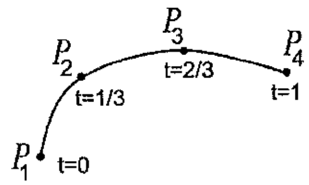
This quiz has 4 questions, for a total of 29 points.

1. Parametric Curves

   (a) (3 points) Develop a cubic parametric curve, $P(t) = TMG$ that interpolates the control points P_1, P_2, P_3 and P_4 as shown below. Do not bother with inverting any matrices.



$$x(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \cdot A$$
$$x_1 = x(0) = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \cdot A$$
$$x_2 = x(\tfrac{1}{3}) = \begin{bmatrix} \tfrac{1}{27} & \tfrac{1}{9} & \tfrac{1}{3} & 1 \end{bmatrix} \cdot A$$
$$x_3 = x(\tfrac{2}{3}) = \begin{bmatrix} \tfrac{8}{27} & \tfrac{4}{9} & \tfrac{2}{3} & 1 \end{bmatrix} \cdot A$$
$$x_4 = x(1) = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \cdot A$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ \tfrac{1}{27} & \tfrac{1}{9} & \tfrac{1}{3} & 1 \\ \tfrac{8}{27} & \tfrac{4}{9} & \tfrac{2}{3} & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot A$$

call this B and define $M = B^{-1}$

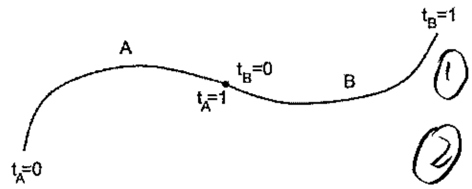
$$G = B \cdot A$$
$$\Rightarrow A = B^{-1} \cdot G$$

$$P(t) = T \cdot A = T \cdot M \cdot G$$

   (b) (2 points) Write expressions for the first and second derivatives of the curve, i.e., $P'(t)$ and $P''(t)$. There is no need to simplify the expressions.

$$P(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \cdot M \cdot G$$
$$P'(t) = \begin{bmatrix} 3t^2 & 2t & 1 & 0 \end{bmatrix} \cdot M \cdot G$$
$$P''(t) = \begin{bmatrix} 6t & 2 & 0 & 0 \end{bmatrix} \cdot M \cdot G$$

   (c) (2 points) Write the constraints that would be needed to connect two parametric cubic curves, A and B, as shown below, with C_2 continuity. Assume a general form for each cubic curve, i.e., $P_A(t) = T_A M_A G_A$ and $P_B(t) = T_B M_B G_B$.



(1) $P_A(1) = P_B(0)$

(2) $P'_A(1) = P'_B(0)$

(3) $P''_A(1) = P''_B(0)$

C_2 continuity requires (1), which then also implies that (1) and (2) are required.

(d) (2 points) A B-spline curve is defined by $P(t) = \underbrace{TM_{BS}}G_{BS}$ where

$$M_{BS} = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}.$$

this product defines the basis functions.

Based on this, give the B-spline basis functions.

$$[t^3 \; t^2 \; t \; 1] \, \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} = [b_1(t) \; b_2(t) \; b_3(t) \; b_4(t)]$$

where
$$\left.\begin{aligned} b_1(t) &= \tfrac{1}{6}(-t^3 + 3t^2 - 3t + 1) \\ b_2(t) &= \tfrac{1}{6}(3t^3 - 6t^2 + 4) \\ b_3(t) &= \tfrac{1}{6}(-3t^3 + 3t^2 + 3t + 1) \\ b_4(t) &= \tfrac{1}{6}t^3 \end{aligned}\right\} \text{note that } \sum_i b_i(t) = 1$$

(e) (1 point) Give the order of parametric continuity, C_n, and geometric continuity, G_n, for Catmull-Rom curves.

Catmull-Rom curves are C_1, which also implies G_1

2. Displays

(a) (2 points) Give the frame rate for (i) recording feature films, and (ii) broadcast television.

(i) 24 fps for recording, with each frame shown twice for playback on a non-digital projector.

(ii) 30 Hz (or, more precisely, 29.97 Hz for NTSC)

(b) (2 points) Aside from the frame rate, give five other attributes for displays that are useful for characterizing displays.

- physical size
- resolution
- brightness
- contrast
- colour gamut, # of colour primaries
- colour depth
- stereo capability
- ... ?

3. Representing rotations

(a) (3 points) Show that the following matrix satisfies (or doesn't satisfy) all the constraints of a rotation matrix.

$$\frac{1}{2}\begin{bmatrix} 1 & -1 & \sqrt{2} \\ \sqrt{2} & \sqrt{2} & 0 \\ -1 & 1 & \sqrt{2} \end{bmatrix}$$
$\quad i \quad j \quad k$

$$|i| = \frac{1}{2}\sqrt{1^2 + (\sqrt{2})^2 + (-1)^2} = \frac{1}{2}\sqrt{4} = 1$$

$|j| = 1$ (similarly)

$|k| = 1$ (similarly

$i \cdot j = 0 : 1(-1) + \sqrt{2}^2 - 1 = 0$

$j \cdot k = 0 : -\sqrt{2} + \sqrt{2} = 0$

$i \cdot k = 0 : \sqrt{2} + -\sqrt{2} = 0$

Then check $\det = +1$

or simply $i \times j = k$

this is also true.

This is a rotation matrix

$\quad i \quad j \quad k$

(1) $\frac{1}{2}(1 \;\; \sqrt{2} \;\; -1)$

(2) $\frac{1}{2}(-1 \;\; \sqrt{2} \;\; 1)$

(1) × (2) $= \frac{1}{4}(2\sqrt{2}, 0, 2\sqrt{2}) = \frac{1}{2}(\sqrt{2}, 0, \sqrt{2})$

(b) (2 points) Compute the quaternion multiplication $a \otimes b$ where $a(w,x,y,z) = (2,0,0,-1)$ and $b(w,x,y,z) = (1,-1,2,0)$. For your reference, the underlying rules of quaternion algebra are: $i^2 = j^2 = k^2 = -1$

$i \cdot j = k, \quad j \cdot i = -k$
$j \cdot k = i, \quad k \cdot j = -i$
$k \cdot i = j, \quad i \cdot k = -j$

$$a \otimes b$$
$$= (2-k) \otimes (1 - i + 2j)$$
$$= 2 - 2i + 4j - k + ki - 2kj$$
$$= 2 - 2i + 4j - k + j + 2i$$
$$= 2 + 5j - k$$

(c) (3 points) The orientation of an object is the result of a 90 degree rotation arond the y-axis followed by a 90 degree rotation around the new x-axis. Express this orientation using: (i) a quaternions, using quaternion multiplication; (ii) in an angle-axis format, as a single rotation; (iii) using XYX Euler angles.
Reminder: The quaternion $(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}\mathbf{k})$ represents a rotation of θ degrees around the axis $\mathbf{k}$.

(i) $q_1 = (\cos(45°) + \sin(45°)j) = \frac{1}{\sqrt{2}}(1+j)$
$q_2 = (\cos(45°) + \sin(45°)i) = \frac{1}{\sqrt{2}}(1+i)$
$q_1 \otimes q_2 = \frac{1}{2}(1+j)(1+i) = \frac{1}{2}(1+i+j+ji) = \boxed{\frac{1}{2}(1+i+j-k)}$

(ii) converting the answer from part (i) back into angle-axis format:
$\frac{1}{2}(1+i+j-k) = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$
$\equiv (\cos\frac{\theta}{2}, \sin\frac{\theta}{2}\vec{k})$
$\Rightarrow \cos\frac{\theta}{2} = \frac{1}{2} \Rightarrow \frac{\theta}{2} = 60°, \theta = 120°$
$\Rightarrow k =$ normalized version of $\langle \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \rangle = \frac{1}{\sqrt{3}}(1,1,-1)$
$\Rightarrow \boxed{Rot(120°, \frac{1}{\sqrt{3}}(1,1,-1))}$

(iii) trivially: $Rot(x, 0°)\, Rot(y, 90°)\, Rot(x, 90°) \Rightarrow XYX = \boxed{0°, 90°, 90°}$

(d) (2 points) List the advantages and disadvantages of using Euler angle representations.
advantages: - minimal representation: only 3 numbers
- widely used
disadvantages: - not a unique representation
- does not interpolate well
- requires conversion to matrices in order to compose successive rotations

(e) (2 points) Compute the mean orientation of the orientations given by $q_A = (1,0,0,0)$ and $q_B = \frac{1}{\sqrt{2}}(-1,-1,0,0)$, as represented by a unit quaternion.

→ compute $q' = 0.5(q_A + q_B)$ then renormalize, as in linear interpolation or spherical linear interpolation.
But first check to see whether it is better to use q_A and q_B, or q_A and $-q_B$, i.e., "go the short way around".
$q_A \cdot q_B = -\frac{1}{\sqrt{2}} = \cos\theta \Rightarrow \theta = 135°$
$q_A \cdot -q_B = \frac{1}{\sqrt{2}} = \cos\theta \Rightarrow \theta = 45°$ → use this. $q' = 0.5(1,0,0,0) + \frac{0.5}{\sqrt{2}}(1,1,0,0)$
$= ($ [illegible] $) \Rightarrow$ then normalize.

4. (3 points) List three general ways to author or create animated motions.

(1) - capture the motion of real people & objects

(2) artists & animators

(3) procedural methods or simulation.