CPSC 426: Computer Animation
Assignment 2

out: Fri February 7, 2014
due: in class Wed February 28, 2014

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name: Solutions

Student Number: ______________________

Question 1	/ 2
Question 2	/ 2
Question 3	/ 2
Question 4	/ 5
Question 5	/ 3
Question 6	/ 6
TOTAL	/ 20

This assignment has 6 questions, for a total of 20 points.

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

① $(a \times b = c)$:
$a_2b_3 - a_3b_2 = c_1$
$a_3b_1 - a_1b_3 = c_2$
$a_1b_2 - a_2b_1 = c_3$
② det = 1
$a_1b_2c_3 + b_1c_2a_3 + a_2b_3c_1 - a_3b_2c_1 - a_2b_1c_3 - a_1b_3c_2 = \phi$.

$M = S \cdot R$

$$\begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & -1 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{bmatrix}$$

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0.5 1.5

1. (2 points) Is the follosing a combination of a rotation matrix and a uniform scale? Why or why not?

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & -sqrt2 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \end{bmatrix}$$

$|i| = |j| = |k| = \sqrt{2}$

$i \cdot j = 0$, $i \cdot k = 0$, $j \cdot k = 0$ } vectors are orthogonal

but: $i \times j = -k$ so this is not a rotation matrix.

$i \times j = k$

2. (2 points) Compute the following quaternion product:
$(2 - 3\mathbf{i} + \mathbf{k}) \otimes (0 + \mathbf{i} - \mathbf{j})$ $= 2i - 2j - 3i^2 + 3ij + ki - kj$
$= 2i - 2j - 3(-1) + 3k + j + i$
$= 3 + 3i - j + 3k$

scaler vector

i j k

3. (2 points) Show that the unit quaternions given by q and $-q$ represent the same orientation, where $q = \langle \cos\frac{\theta}{2}, \sin\frac{\theta}{2}\vec{\mathbf{k}} \rangle$, representing a rotation of θ around the axis $\vec{\mathbf{k}}$.

Let q = quaternion for Rot(k, θ)

Let q' = quaternion for Rot$(k, \theta + 360°)$:

$q' = \langle \cos\left(\frac{\theta + 360}{2}\right), \sin\left(\frac{\theta+360}{2}\right)\vec{k} \rangle$
$= \langle \cos\left(\frac{\theta}{2} + 180°\right), \sin\left(\frac{\theta}{2} + 180°\right)\vec{k} \rangle$
$= \langle -\cos\left(\frac{\theta}{2}\right), -\sin\left(\frac{\theta}{2}\right)\vec{k} \rangle = -q$

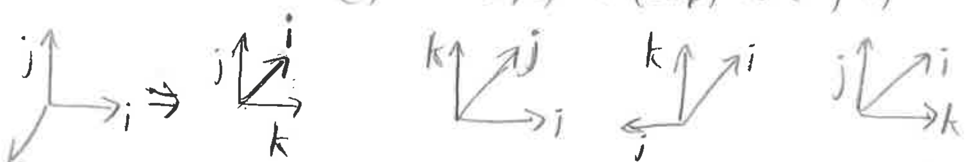
not a good direction:
$q \otimes -q = \langle \cos\frac{\theta}{2}, \sin\frac{\theta}{2}\vec{k} \rangle \cdot \langle -\cos\frac{\theta}{2}, -\sin\frac{\theta}{2}\vec{k} \rangle$

4. (5 points) Express Rot$(y, 90°)$ in the following representations: (a) rotation matrix; (b) quaternion; (c) XZX Euler angles.

$-\cos^2\frac{\theta}{2} - 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\vec{k} + \sin^2\frac{\theta}{2} = \sin\frac{\theta}{2}\left(\sin\frac{\theta}{2} - \cos\frac{\theta}{2}\vec{k}\right)$
$-\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}\right.$

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ (b) $q = \langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0 \rangle$

(c) Rot(x, θ) Rot(z, ϕ) Rot(x, α)



Rot$(x, -90°)$ Rot$(z, 90°)$ Rot$(X, +90°)$

5. (3 points) Using quaternion multiplication, show that
Rot$(z, 90°)$Rot$(x, -90°)$Rot$(z, -90°)$ = Rot$(y, -90°)$

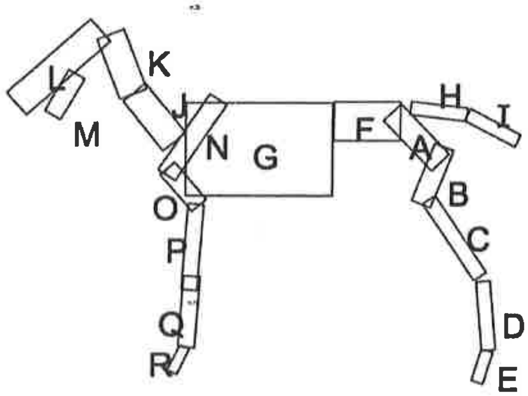
$q_1 \otimes q_2 \otimes q_3$

$\frac{1}{\sqrt{2}}(1 + k) \otimes \frac{1}{\sqrt{2}}(1 - i) \otimes \frac{1}{\sqrt{2}}(1 - k)$

$= \frac{1}{2}(1+k)(1-i)$
$= \frac{1}{2}(1 - i + k - ki)$
$= \frac{1}{2}(1 - i - j + k) \otimes \frac{1}{\sqrt{2}}(1 - k)$
$= \frac{1}{2\sqrt{2}}(1 - i - j + k - k + ik + jk - k^2)$
$= \frac{1}{2\sqrt{2}}(1 - i - j + k - k - j + i + 1)$

$= \frac{1}{2\sqrt{2}}(2 - 2j)$
$= \frac{1}{\sqrt{2}}(1 - j)$
$=$ Rot$(y, -90°)$

~~6.~~ (6 points) Consider the articulated skeleton model of a horse, as shown below.



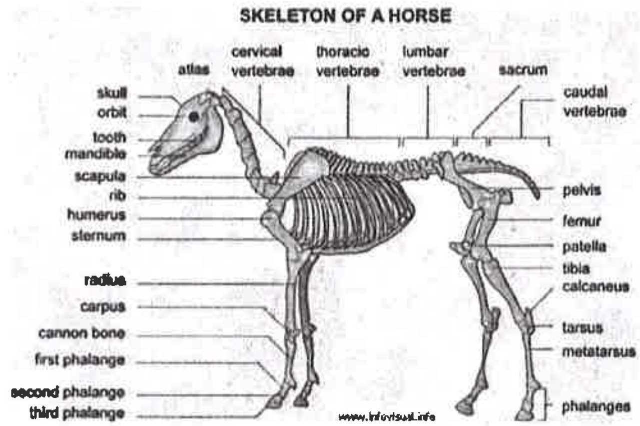
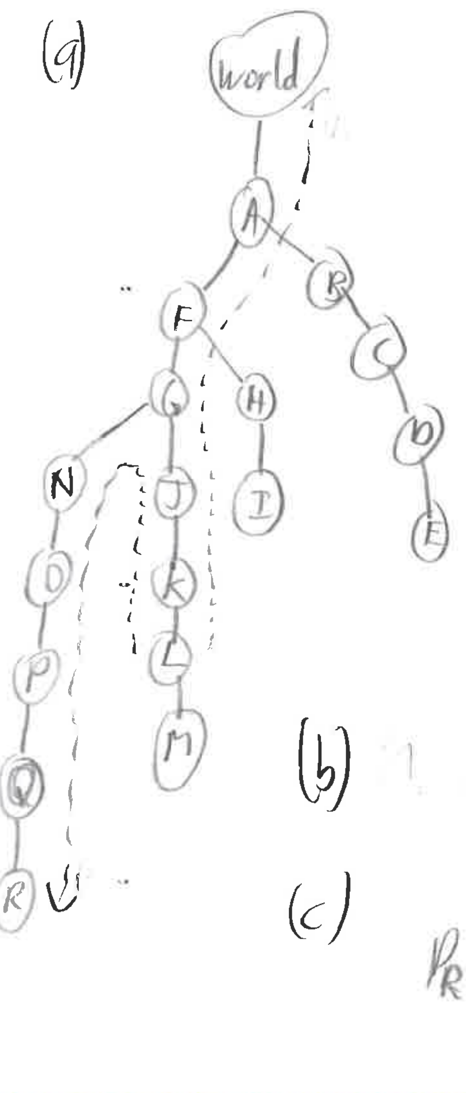


Image from: http://www.infovisual.info/02/072_en.html

(a) [2] Sketch a scene graph for all the given links for the horse. Label the links in your scene graph with the letters used in the above diagram. Assume that link A is the root link, i.e., that its parent link is the world coordinate frame.

(a)



each error : -0.5

(b) $P_{world} = \overbrace{M_A M_F M_G M_J M_K M_L}^{M_{L\to world}} P_L$

written error -0.5

comprehensive error -1 / -2

(c) $P_R = \overbrace{M_R^{-1} M_Q^{-1} M_P^{-1} M_O^{-1} M_N^{-1} M_J M_K M_L}^{M_{L\to R}} P_L$

(b) [2] Develop an expression for the transformation matrix that would be used to render the head of the horse, i.e., that takes points from frame L to the world frame. Assume that the transformation matrix M_Z that is associated with link Z takes a point from its coordinate frame to to its parent frame.

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(c) [2] Develop an expression for the transformation matrix that takes a point in the head coordinate frame and expresses it in the coordinate frame of the hoof of the front leg, i.e., frame R.

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