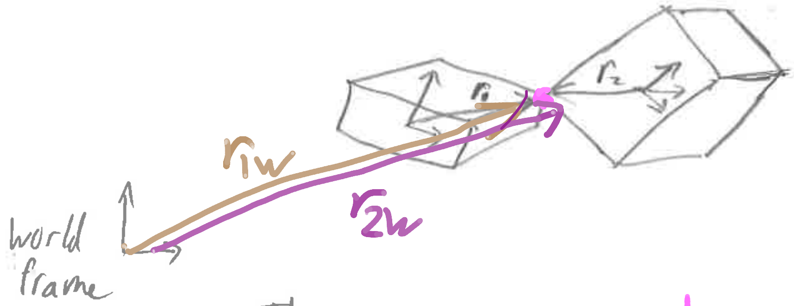
Simulation of two rigid bodies with a constraint



This requires: constraint force : F_c

3 ext

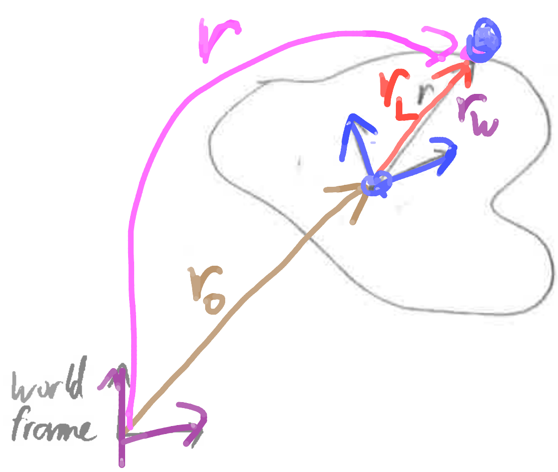
$$r_{1w} - r_{2w} = 0$$

$$\dot{r}_{1w} - \dot{r}_{2w} = 0$$

$$\boxed{\ddot{r}_{1w} - \ddot{r}_{2w} = 0}$$

3 extra equations

Computing the acceleration of a point on a rigid rotating



$$r_w = R r_L$$

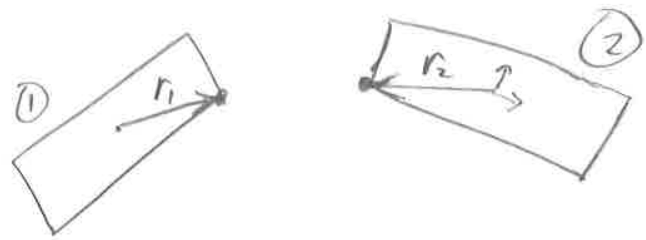
$$r = r_0 + r_w$$

$$r = r_0 + R r_L$$

$$\begin{aligned} \dot{r} &= \dot{r}_0 + \dot{R} r_L \\ &= \dot{r}_0 + \tilde{\omega}(R r_L) \\ &= \dot{r}_0 + \omega \times (R r_L) \\ &= \dot{r}_0 + \omega \times r_w \end{aligned}$$

$$\ddot{r} = \underbrace{\ddot{r}_0}_{\text{linear acc}} + \underbrace{\dot{\omega} \times r_w}_{\text{tangential acc}} + \underbrace{\omega \times (\omega \times r_w)}_{\text{centripetal acc}}$$

Newton - Euler Equations for two blocks with a point-to-point



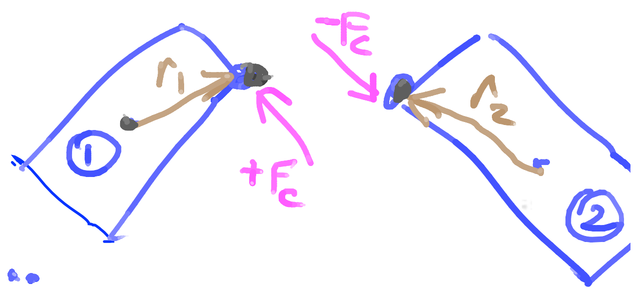
Block ① Newton

Block ① Euler

Block ② Newton

Block ② Euler

Constraint equation



Block ① Newton

$$\Sigma F = m_1 \ddot{x}_1$$

$$F_c + m_1 \vec{g} = m_1 \ddot{x}_1$$

$$-F_c + m_1 \ddot{x}_1 = +m_1 g$$

Simila

Block ① Euler

$$\Sigma \tau = I_1 \dot{w}_1 + w_1 \times (I w_1)$$

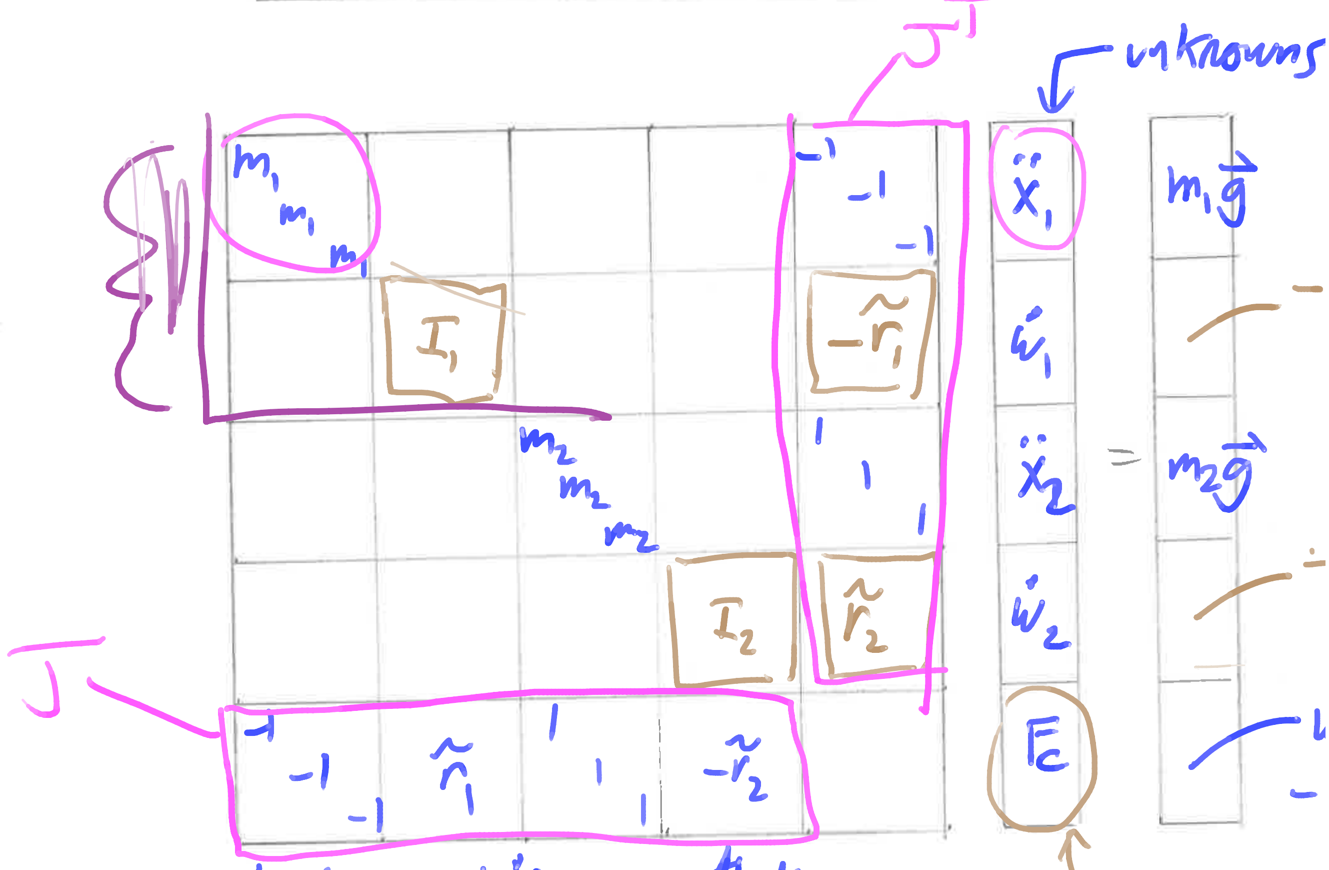
$$r_1 \times F_c = I_1 \dot{w}_1 + w_1 \times (I w_1)$$

$$I_1 \dot{w}_1 - \tilde{r}_1 F_c = -w_1 \times (I w_1)$$

Constraint eqn: $\ddot{x}_1 + \dot{w}_1 \times r_1 + w_1 \times (w_1 \times r_1) = \ddot{x}_2 + \dot{w}_2 \times r_2$

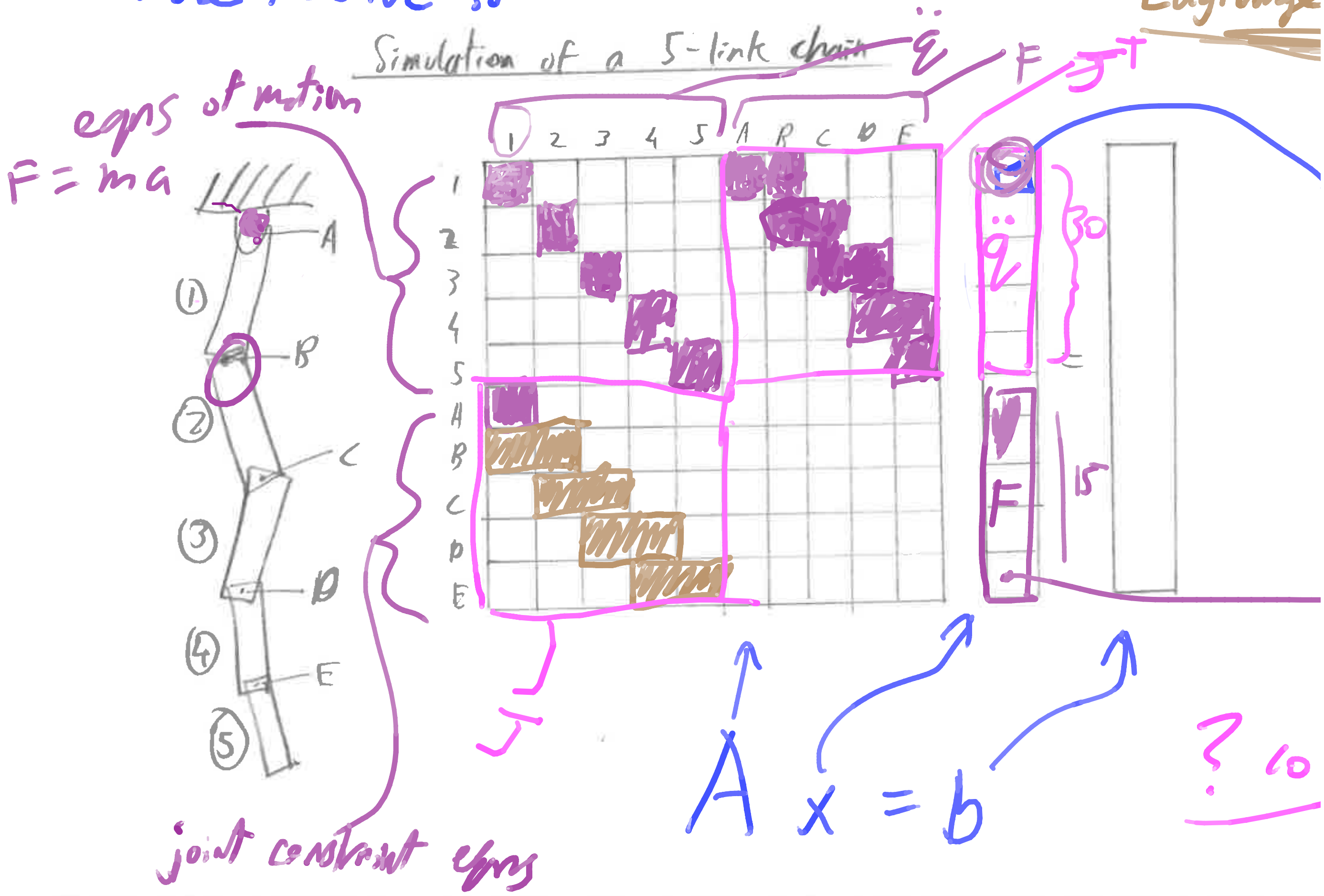
$$\ddot{x}_1 - \tilde{r}_1 \dot{w}_1 - \ddot{x}_2 + \tilde{r}_2 \dot{w}_2 = w_2 \times (w_2 \times r_2)$$

Newton-Euler Equations in Matrix Form

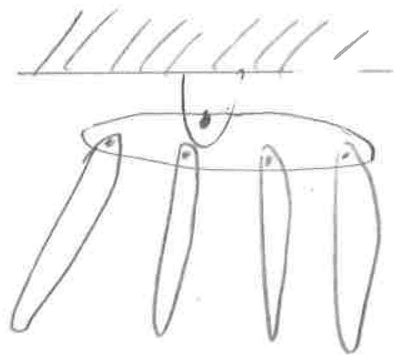


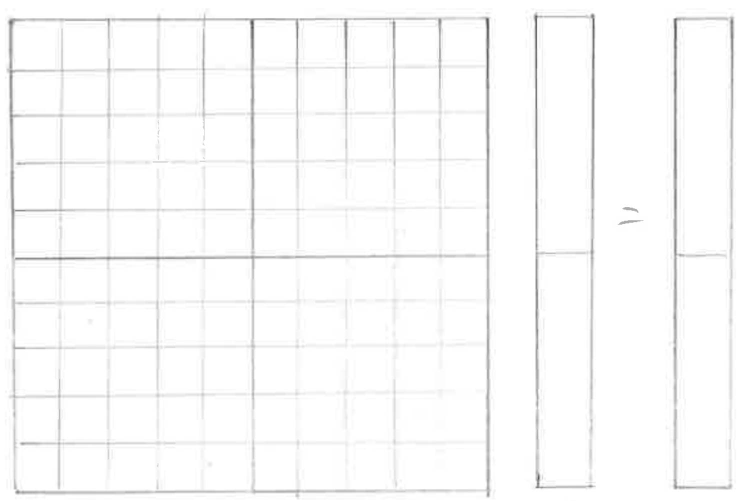
→ use iterative solution methods.

Simulation of a 5-link chain



Another Example





Efficient Solutions that exploit the sparsity of these matrices:

"Linear Time Dynamics Using Lagrange Multipliers"
by David Baraff, SIGGRAPH 1996