## Name:

## Student ID:

1) Why is the frame rate for film different from the rate at which it's projected?

Film runs at 24 fps, which is enough to convince viewers of smooth motion: faster would be an unnecessary expense. However, flashing frames with a projector at $24 f p s$ is too slow for flicker fusion, especially in peripheral vision, so it is projected at $48 f p s$ (each frame shown twice) to avoid apparent flicker.
2) Why is it difficult to convincingly integrate stop-motion animation into real footage?

In stop-motion every frame is a separate photograph of a still scene, and thus doesn't show motion blur. Real footage of moving scenes has motion blur: this is very apparent when combining the two.
3) Write down the lerp at time $t$ between control points $\left(2, f_{0}\right)$ and $\left(10, f_{1}\right)$.

$$
f(t)=\left(\frac{10-t}{8}\right) f_{0}+\left(\frac{t-2}{8}\right) f_{1}
$$

4) Write down a formula for the $i$ 'th Bernstein basis polynomial of degree $n$.

$$
B_{i, n}(s)=\binom{n}{i}(1-s)^{n-i} s^{i}
$$

5) Why does a Forward Kinematic skeleton have to be structured as a tree (or forest) instead of a more general directed graph?
Every node's frame must be related by a joint transformation to the frame of a unique parent node (or directly to the world space, with no parent): if there was more than one parent, the two joints could be in conflict leading to an unacceptable ambiguity. This is equivalent to requiring the graph be a tree or forest.
6) How could you model the joint connecting an eyeball's frame to the head's frame, and in particular, how many DOFs do you need?
A reasonable model has two rotational DOF: fixing the centre of the eyeball to a constant position within the head and allowing rotation about the $x$ axis (up-down) and $y$ axis (left-right).
7) Give the details on performing linear blend skinning for a mesh vertex $i$ with rest position $\vec{p}_{i k}$ with respect to bone $k$, blend weight $w_{i k}$ for bone $k$, and transformation $T^{W \leftarrow k}$ from bone $k$ to world space, for $k=1, \ldots n$.
Each bone provides a guess on where it thinks the vertex should be based on its own frame: $T^{W \leftarrow k} \vec{p}_{i k}$. The actual position of the vertex is a weighted average of the bones' guesses using the blend weights:

$$
\vec{p}_{i W}=\sum_{k=1}^{n} w_{i k}\left[T^{W \leftarrow k} \vec{p}_{i k}\right]
$$

8) Suggest a context where blend shapes would be more useful than linear blend skinning.

Facial expressions. The face has almost no natural bones, no natural hierarchy, and instead is a complex web of interconnecting muscles attached to the skull, each other, and the skin - so FK and linear blend skinning is a bad fit. Blend shapes are much more intuitive for the artist.
9) Define and describe how matchmove and inverse kinematics are related.

Matchmove: solving for the transformation between world space and camera space given constraints on where markers have to lie.

Name:

## Student ID:

Inverse Kinematics: solving for the transformations between bones in a skeleton given constraints on where end-effectors have to lie.

Both are inverse problems, solving for transformations given geometric constraints on the transformations, and may use the same optimization methods.
10) Why is the problem $\min _{x, y}(x-y-1)^{2}$ (solve for $x$ and $y$ ) ill-posed, and how could you make it well-posed?
There are infinitely many solutions: any pair $(x, y)$ where $x=y+1$ achieves the minimum value of 0 . A well-posed problem must have a unique solution instead. This could be regularized to make it well-posed, for example adding a term that penalizes larger solutions:

$$
\min _{x, y}(x-y-1)^{2}+10^{-6}\left(x^{2}+y^{2}\right)
$$

This has the unique solution $x=1 /\left(2+10^{-6}\right), y=-1 /\left(2+10^{-6}\right)$.

