Research Advertisement

Mark Greenstreet, CpSc 421, Term 1, 2008/09

- Today's NP-Completeness Example: SUBSET-SUM
- Research Advertisement
  - Parallel Computing
  - Circuit Verifcation

SUBSET SUM

- Instance:
  - Let S be a set,
  - Let $w : S \to \mathbb{Z}^+$ be a function that gives the “weight” of elements of s.
  - Let t be an integer.
- Question: Is there a set $C \subseteq S$ such that the sum of the weights of the elements of C is equal to t?
- SUBSET SUM is NP-complete
  - It is easy to see that SUBSET SUM in NP, proposing a set C suffices as a certificate.
    - Such a subset is shorter than the original input, thus its size is polynomial in the length of the input.
    - Checking that $C \subseteq S$ and that $\sum_{m \in C} m = t$ are straightforward and polynomial time.
  - To show that SUBSET SUM is NP hard, we reduce one-in-three 3SAT to SUBSET SUM.

SUBSET SUM: Details

Verifying the Reduction

PARTITION is NP-Complete

- Problem instance: a finite set S and a weight function $w : S \to \mathbb{Z}^+$.
- Question: Can S be partitioned into 2 disjoint sets, S_1, S_2 such that $\sum_{s \in S_1} s = \sum s \in S_2 s$?
- PARTITION is NP-complete. Proof: by reduction from SUBSET SUM.

Dynamic Programming

- If there is some subset of S whose sum equals t, we can perform that sum in order of increasing weights of the elements. Let $w_1, \ldots, w_m$ be this sequence of weights.
- This leads to a dynamic programming algorithm for solving SUBSET SUM.

```
SubsetSum(Set<int> s, int t) {
    int[1 ... t] x; /* initially all elements set to m + 1 */
    for int i = 1...t do {
        for j = 1...m do {
            if((w_j == i) | ((w_j < i) & (x[i-w_j] < w_j))) {
                x[i] = j;
                break; /* for m */
            }
        }
    }
    return(x[t] ≤ m);
}
```

This algorithm runs in $O(t^2)$ time!

Weak vs. strong NP completeness

- A numerical problem has a pseudo-polynomial time complexity if it can be decided in time that is a polynomial in the values of the numbers occuring in the input.
  - SUBSET SUM has a pseudo-polynomial decision procedure.
- For a numerical problem with input I, let $Length(I)$ be the number of symbols in I and $Max(I)$ be the largest (in absolute value) integer encoded by I.
- If there is a polynomial p such that a problem, X, is NP-complete when restricted to inputs I with

$$Max(I) \quad \leq \quad p(Length(I))$$

  then we say that X is strongly NP-complete.
- Less formally, X is strongly NP-complete if there is no pseudo-polynomial decision procedure for X (unless P $=$ NP).

3-Partition is Strongly NP-Complete

- Problem instance: a finite set S of $3m$ positive integers, a positive integer b, such that each $s \in S$ satisfies $b/4 < s < b/2$, and such that $\sum_{s \in S} s = mb$.
- Question: Can S be partitioned into m disjoint sets, $S_1, S_2, \ldots S_m$ such that for every $1 \leq i \leq m$, $\sum_{s \in S_i} s = b$?
- Note: by the constraints on S, each of the S_i must have exactly three elements.

The material here on strong NP completeness is based on chapter 4 of Garey & Johnson, "Computers and Intractability."

Multiprocessor Scheduling

- Problem instance:
  - a set, T of tasks,
  - an integer-valued function $l(t)$ that for each $t \in T$ says how long task t takes to run,
  - a positive integer, p, the number of processors,
  - a positive integer d, the deadline.
- Question: Can tasks be assigned to processors such that all tasks complete by the deadline?
- NP-completeness:
  - Multi-processor scheduling is NP-complete for any $p \geq 2$: proof by reduction from PARTITION.
  - Pseudo-polynomial time decision procedures exist for any fixed p (but the degree of the polynomial grows with p).
  - Multi-processor scheduling is NP-complete in the strong sense when p is allowed to be arbitrary: proof by reduction from 3-PARTITION.

Parallel Computing

Areas of interest:

- Parallel architectures
- Applications of parallel computing
- Energy-time trade-offs in computation
- Compilers and other system software for parallel computation

Superscalar Architectures: old ILP

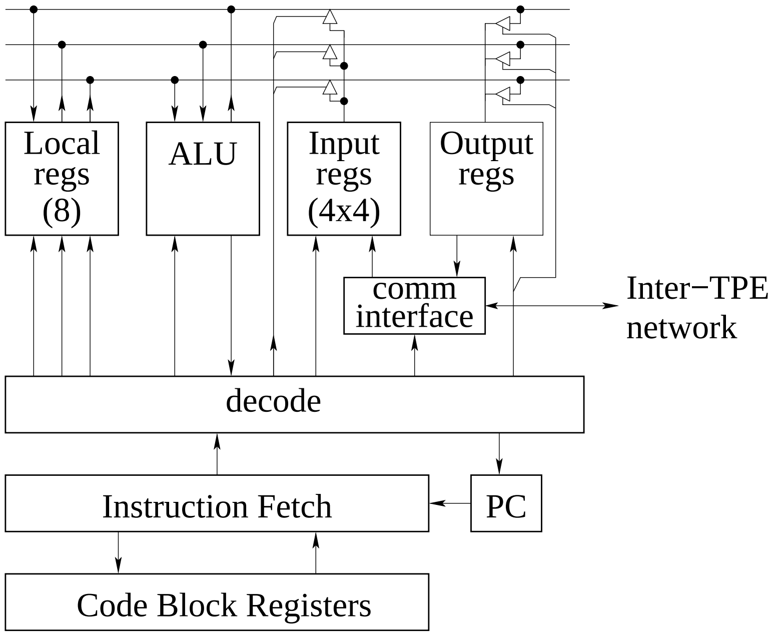
- Fetch several (~ 4) instructions per cycles.
- Rename registers:
  - A logical register to physical register mapping, similar to virtal memory.
  - Keeps track of data dependencies between instructions.
- Send renamed instructions to issue queues of functional units.
  - When an instruction has its operands, it can execute.
  - Because the machine has multiple functional units, it can execute multiple instructions per cycle.
- Key issues:
  - Need to wait to commit an instruction until all previous instructions commit.
  - To find instructions that can execute in parallel, need to fetch beyond pending branches.
    - This requires branch speculation.

Merging on a Superscalar

```
while(x < xtop && y < ytop) {
    if(*x < *y) *z++ = *x++;
    else *z++ = *y++;
}
```

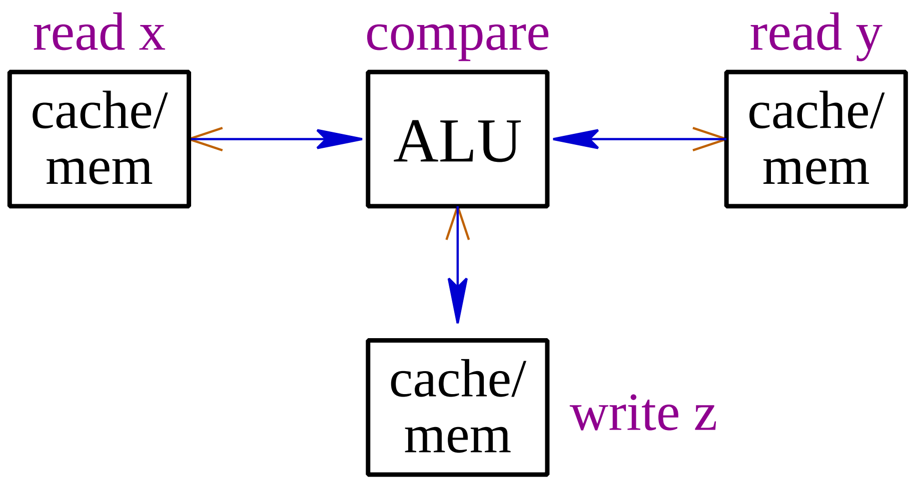
- Data dependent branch in each iteration
- Any branch predictor wrong 50% of time for random data
- High mispredict penalties leads to low perofrmance

Tiny Processing Elements (TPEs)



- Divide a “core” into many small processors
- Each has its own instruction store and fetch
- Heterogeneous in the small,
  - homogeneous in the large(?)
- Communication through registers with FIFO semantics

Merging on a TPE cluster



- Separate cache-TPEs fetch x and y streams.
- Another TPE merges the two streams.
- A fourth TPE writes the result back to cache/memory.
- The fetch and store TPEs make progress regardless of the branch outcome for the compare TPE.

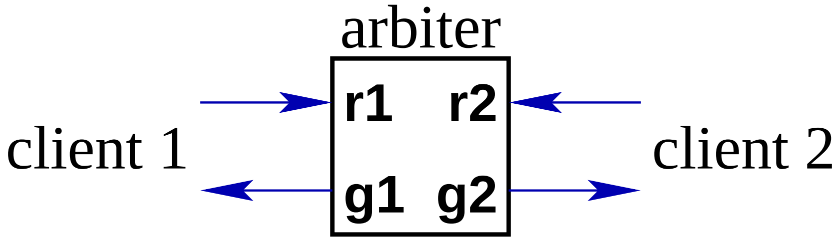
TPE Research Questions

- How to take advantage of multiple instruction streams for executing a single thread.
- How to write a compiler for TPEs:
  - Compilers for traditional (C, C++, Java) languages.
  - Compilers for languages with explicit parallelism.
- On-chip network design.
- Power vs. speed trade-offs.
- …

Why do Circuit-Level Verification?

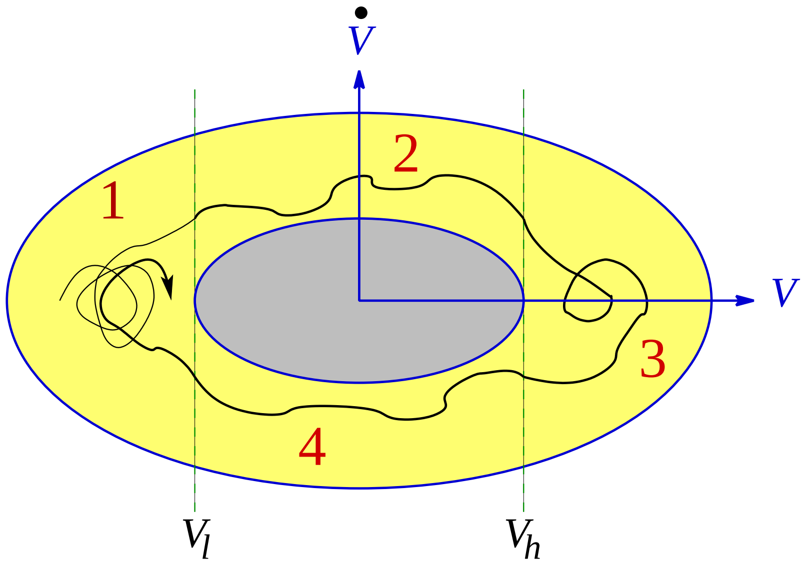
- Digital design has become relatively low error:
  - Systematic design flows.
  - Lots of simulation.
  - Equivalence checking.
  - Model checking.
- Circuit-level bugs remain a problem:
  - SPICE is still the main validation tool, and it doesn't scale.
  - Deep-submicron circuit effects undermine digital abstractions.
  - Hard/impossible to simulate bugs.

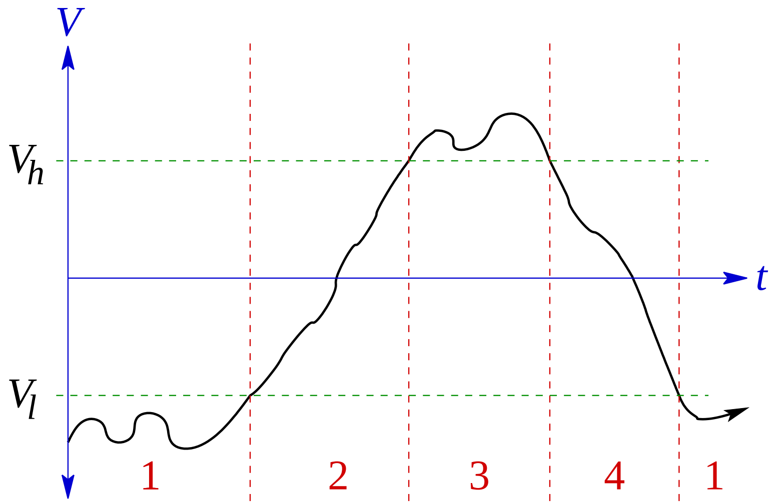
Arbiters



- Specification
  - Initially: $\neg r_1 \wedge \neg r_2 \wedge \neg g_1 \wedge \neg g_2$.
  - Assume: $\Box r_i \, U \, g_i$, $\Box \neg r_i \, U \, \neg g_i$.
  - Guarantee:
    - Handshake: $\Box \neg g_i \, U \, r_i$, $\Box g_i \, U \, r_i$.
    - Mutual Exclusion: $\Box \neg (g_1 \wedge g_2)$.
    - Liveness: $\Box (r_1 \oplus r_2) \Rightarrow \Diamond (g_1 \oplus g_2) \vee (r_1 \wedge r_2)$, $\Box \neg r_i \Rightarrow \Diamond \neg g_i$.
      Note: because metastability is unavoidable, no arbiter can guarantee $\Box (r_1 \wedge r_2) \Rightarrow \Diamond (g_1 \vee g_2)$.
- Why Verify an Arbiter?
  - Exercise in modeling concurrent events from the environment.
  - Requires handling a non-trivial circuit behavior: metastability.

Specifying an Arbiter

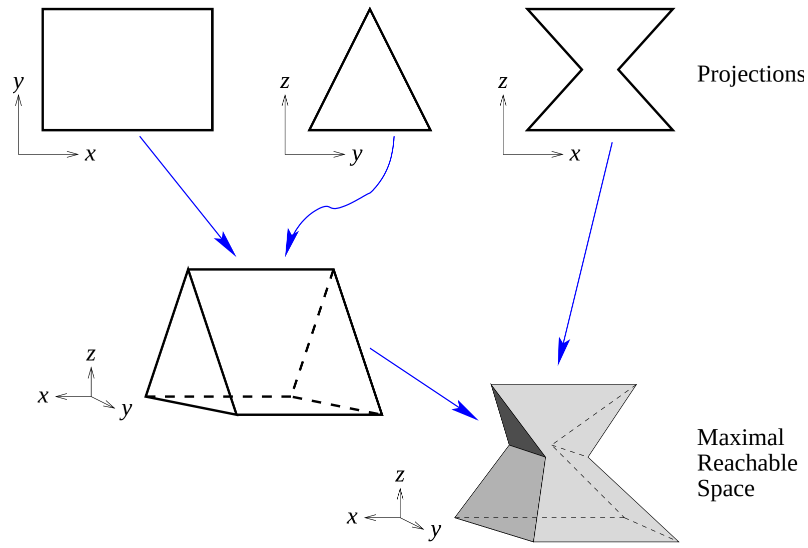




- Specifying signal behavior – Brockett's annulus:
  - Region 1 represents a logical low signal. The signal may wander in a small interval.
  - Region 2 represents a monotonically rising signal.
  - Region 3 represents a logical high signal.
  - Region 4 represents a monotonically falling signal.
  - Brockett's annulus allows entire families of signals to be specified.

Coho: Reachability Using Projections

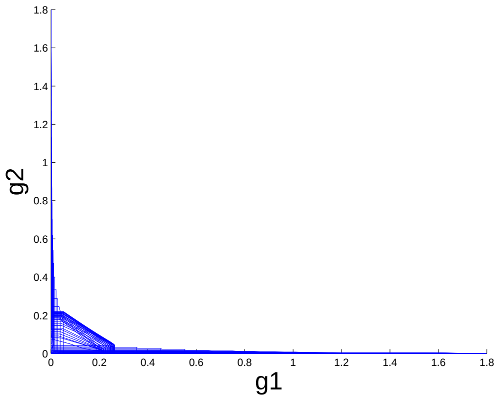
- Coho projects high dimensional polyhedron onto two-dimensional subspaces.
- A projectagon is the intersection of a collection of prisms, back-projected from the projection polygons.
- Coho computes reachable sets by integrating over a series of timesteps:
  - A bounding projectagon is obtained by moving each face forward in time.
  - Projectagon faces correspond to projection polygon edges; thus, Coho works on one edge at a time.



from: http://pond.dnr.cornell.edu/

Results

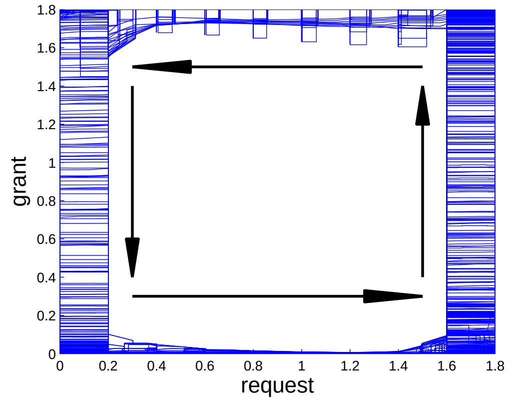
- Safety Properties
  - Mutual Exclusion
  - Handshake Protocol
  - Brockett Annuli
- Liveness Properties



Mutual Exclusion

Results

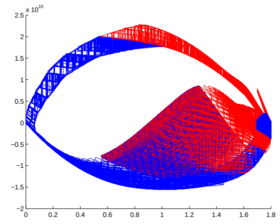
- Safety Properties
  - Mutual Exclusion
  - Handshake Protocol
  - Brockett Annuli
- Liveness Properties

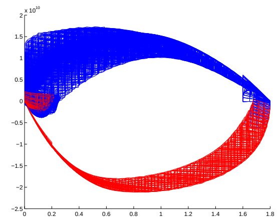


Handshake

Results

- Safety Properties
  - Mutual Exclusion
  - Handshake Protocol
  - Brockett Annuli
- Liveness Properties





$\dot{x}$ vs. x $\dot{g}$ vs. g.

Brockett Annuli

Results

- Safety Properties
  - Mutual Exclusion
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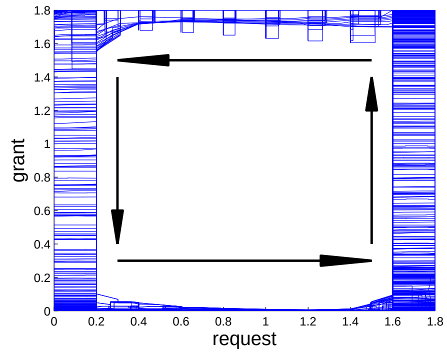
  - Brockett Annuli



Handshake

- Liveness Properties:
  - Initialization: stable within 200ps
  - Uncontested Requests: grant the client within 350ps
  - Contested Requests: metastability within hyper-rectangle

$$\begin{array}{lll} r_1 \in B_3 & x_1 \in [0.55, 1.3] & g_1 \in B_1 \\ r_2 \in B_3 & x_2 \in [0.55, 1.3] & g_2 \in B_1 \end{array}$$

  - Reset: withdraw grants within 270ps
  - Fairness: grant the other client within 420ps

The last slide

- Dec. 1: HW 11 due at 4pm.
- Dec. 1, 3, 5: office hour from 1-2pm.
- Dec. 6: final exam, 3:30-6:30pm, CHBE 103

Thanks to all of you for a good term! ☺