CpSc 421 Homework 6 **Due: Oct. 24, 4pm**

1. (**50 points**) For each language below, determine whether or not the language is context-free, and justify your answer. For example, if the language **is** context-free, you can give a CFG or PDA for the language. If it is not context-free, you can use the pumping lemma for CFLs and/or other properties of CFLs to prove it. You may use relationships to other languages that we have shown to be context-free or not, and you can use the results from previous parts of this problem when solving subsequent parts.

   (a) **(10 points)** $A_1 = \{w \in \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}^* \mid \exists x, y \in \{a, \mathsf{b}\}^*.\ (w = x\#y) \wedge (x \text{ is a substring of } y)\}$

   (b) **(10 points)** $A_2 = \{w \in \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}^* \mid \exists i, j \in \mathbb{Z}^{\geq 0}.\ w = \mathsf{a}^i \mathsf{b}^i c^j$.

   (c) **(10 points)** $A_3 = \{w \in \{\mathsf{a}, \mathsf{b}, \mathsf{c}, \mathsf{d}\}^* \mid \exists i, j \in \mathbb{Z}^{\geq 0}.\ w = \mathsf{a}^i \mathsf{b}^i \mathsf{c}^j \mathsf{d}^j\}$.

   (d) **(20 points)** $A_6 = \overline{E}$, where $E = \{w \in \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}^* \mid \exists n \in \mathbb{Z}^{\geq 0}.\ w = \mathsf{a}^n \mathsf{b}^n \mathsf{c}^n\}$.

2. (**20 points**) Let $G = (V, \Sigma, R, S)$ be a CFG in Chomsky normal form. Show that if G generates any string, w, with $|w| > 2^{|V|-1}$, then $L(G)$ is an infinite set.

3. **(20 points)** Draw the state diagram for a Turing machine that recognizes the language of all strings in $\{\mathsf{a}\}^*$ for which the number of a's in the string is a perfect square.