1. **(20 points** $+10$ ***points extra credit*****)** Write regular expressions that generate each of the languages below. For each language, the alphabet, Σ, is $\{\mathtt{0},\mathtt{1}\}$.

   (a) $A_1 = \{s \mid s$ contains the substring `1011`$\}$.

   (b) $A_2 = \{s \mid |s| = 5m + 7n$ with $m, n \in \mathbb{N}\}$.

   (c) $A_3 = \{s \mid s$ contains an even number of `0`'s$\}$.

   (d) $A_4 = \{s \mid s$ contains an odd number of `1`'s$\}$.

   (e) (*10 points extra credit*):
   $A_5 = \{s \mid s$ contains an even number of `0`'s and an odd number of `1`'s$\}$.

2. **(30 points)** In the problems below, let R_1, R_2, ... be arbitrary regular expressions over an arbitrary finite alphabet. For each proposed identity, either prove it, or give a counter-example. Two are valid identities for which a correct proof is worth 10 points; two are not valid identities for which a counter-example is worth 5 points.

   (a) $R_1 \cup \epsilon = R_1$.

   (b) $R_1 R_1^* = R_1^* R_1$.

   (c) $R_1 \cdot (R_2 \cup R_3) = (R_1 \cdot R_2) \cup (R_1 \cdot R_3)$.

   (d) $R_1 \cup (R_2 \cdot R_3) = (R_1 \cup R_2) \cdot (R_1 \cup R_3)$.

3. **(20 points)** For any language, A, let

   $$A^{\mathcal{R}} = \{s \mid s^{\mathcal{R}} \in A\}$$

   where $s^{\mathcal{R}}$ is the *reverse* of s as defined in homework 0:

   $$\begin{array}{rcl} \epsilon^{\mathcal{R}} & = & \epsilon \\ (x \cdot c)^{\mathcal{R}} & = & c \cdot x^{\mathcal{R}} \end{array}$$

   Prove that if A is regular, then so is $A^{\mathcal{R}}$.

4. **(40 points)**: For each language below, determine whether or not the language is regular. If it is regular, draw a DFA that accepts it and write a *short* explanation of how your DFA works. If it is not regular, provide a proof. For each language, the alphabet, Σ, is $\{\mathtt{0},\mathtt{1}\}$. The notation $\#\mathtt{0}(s)$ refers to the number of $\#\mathtt{0}$'s in s, and $\#\mathtt{1}(s)$ refers to the number of $\#\mathtt{1}$'s.

   (a) $B_1 = \{s \mid s$ contains an even number of `0`'s and an odd number of `1`'s$\}$.

   (b) $B_2 = \{s \mid \#\mathtt{1}(s) = k * \#\mathtt{0}(s)$ for some $k \in \mathbb{N}\}$.

   (c) $B_3 = \{s \mid (|\#\mathtt{0}(s) - \#\mathtt{1}(s)| \bmod 3) = 0\}$.

   (d) $B_4 = \{s \mid (|\#\mathtt{0}(s) - \#\mathtt{1}(s)| \bmod 3) = 1\}$.