

1. **(15 points)** Let  $\Sigma = \{a, b, c\}$ . Figure 1 depicts two finite state machines that read Let  $L_a$  and  $L_b$  denote the languages recognized by DFA (a) and DFA (b) respectively.
- (a) **(6 points)** For each of  $L_a$  and  $L_b$  list three strings in  $\Sigma^*$  that are in the language and three strings in  $\Sigma^*$  that are not in the language.
- (b) **(3 points)** Write a short, English description of language,  $L_a$ .
- (c) **(6 points)** Write a short, English description of language,  $L_b$ .

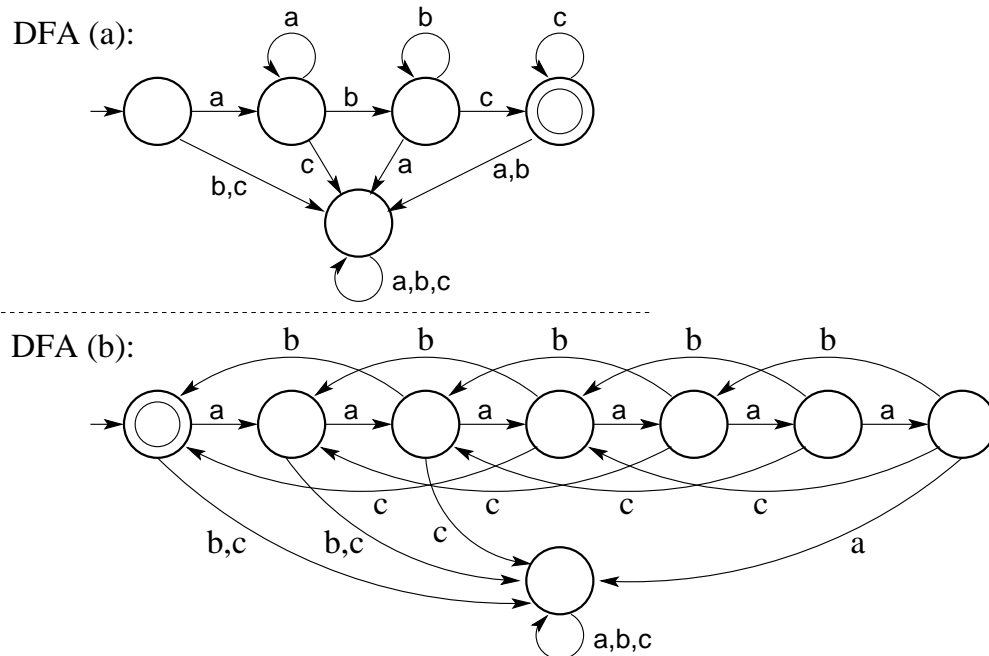


Figure 1: Finite state machines for question 1

2. **(15 points)** (inspired by Sipser exercise 1.6 (p. 84): Give state diagrams of DFAs recognizing the following languages. For each language, the alphabet is  $\{0, 1\}$ .
- (a)  $\{w \mid w \text{ begins and ends with the same symbol}\}$ .
- (b)  $\{w \mid w \text{ contains three consecutive 1s}\}$ .
- (c)  $\{w \mid w \text{ contains neither the substring } 010 \text{ nor the substring } 101\}$ .
3. **(20 points)** (inspired by Sipser exercise 1.7 (p. 84): Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. For each language, the alphabet is  $\Sigma$  with  $\Sigma = \{0, 1\}$ .
- (a) The set of strings  $w$  that end with the substring 010:

$$\{w \mid \exists x \in \Sigma^*. w = x010\}$$

Use four states.

(b) The set of strings  $w$  that contain the substring 010:

$$\{w \mid \exists x, y \in \Sigma^*. w = x010y\}$$

Use four states.

(c) The set of strings  $w$  that can be written as  $x_1 \cdot x_2 \cdot \dots \cdot x_k$  for some  $k \geq 0$ , with each  $x_i$  is an element of

$$\{01, 10, 001, 0011\}$$

Use eleven states.

(d) The set of strings whose length is a multiple of three plus a multiple of five:

$$\{w \mid \exists m, n \in \mathbb{N}. |w| = 3m + 5n\}$$

Use eight states.

4. (20 points): Let  $\Sigma = \{0, 1\}$ . Let  $L_k$  be the set of all strings whose  $k^{\text{th}}$  from end symbol is a 1:

$$L_k = \{w \in \Sigma^* \mid \exists x \in \Sigma^*, y \in \Sigma^{k-1}. w = x1y\}$$

In the September 12 notes, we claimed that any DFA that recognizes  $L_k$  must have at least  $2^k$  states. Prove this claim.

5. (20 points, extra credit): In this exercise, you will prove the equivalence of NFA acceptances as defined in Sipser and the formulation that I gave in class. This problem statement is long because I first summarize both Sipser's and my definitions of acceptance. If you are comfortable with both, you can skip to the end of the problem statement where I ask you to prove the equivalence of the two formulations.

Let  $N_{mrg} = (Q, \Sigma, \Delta, q_0, F)$  be an NFA as defined in the September 12 notes.

$Q$  is a finite set of states;

$\Sigma$  is a finite alphabet;

$\Delta \subseteq Q \times \Sigma_\epsilon \times Q$  is the transition relation;

$q_0 \in Q$  is the initial state; and

$F \subseteq Q$  is the set of accepting states.

where  $\Sigma_\epsilon = \Sigma \cup \{\epsilon\}$ .

Here are the formulas that we used to define NFA acceptance (see the slides for explanations).  $close_\epsilon(q)$  is the subset of  $Q$  where  $p \in close_\epsilon(q)$  iff

$$\begin{aligned} p &= q \\ \exists q' \in close_\epsilon(q). (q', \epsilon, p) &\in \Delta \end{aligned}$$

We extended  $close_\epsilon$  to sets as

$$close_\epsilon(G) = \bigcup_{q \in G} close_\epsilon(q)$$

Then we defined:

$$\begin{aligned} step(q, c) &= close_\epsilon(\{q' \mid (q, c, q') \in \Delta\}), & c \in \Sigma \\ step(G, c) &= \bigcup_{q \in G} step(q, c), & G \subseteq Q, c \in \Sigma \\ \Delta(G, \epsilon) &= close_\epsilon(G), & G \subseteq Q \\ \Delta(G, x \cdot c) &= step(\Delta(G, x), c), & G \subseteq Q, x \in \Sigma^*, c \in \Sigma \end{aligned}$$

Finally, we said that  $N_{mrg}$  accepts  $s$  iff

$$\Delta(\{q_0\}, s) \cap F \neq \emptyset$$

Now, for Sipser's version. Given  $N_{mrg} = (Q, \Sigma, \Delta, q_0, F)$  as described above, let  $N_{ms} = (Q, \Sigma, \delta, q_0, F)$  with  $\delta : Q \times \Sigma_\epsilon \rightarrow 2^Q$  where  $2^Q$  is the power set of  $Q$  and

$$\delta(q, c) = \{q' \mid (q, c, q') \in \Delta\}$$

Sipser says that NFA  $N_{ms}$  accepts string  $s$  iff we can find  $y_1, y_2, \dots, y_m \in \Sigma_\epsilon$  and  $r_0, r_2, \dots, r_m \in Q$  such that

- $s = y_1 \cdot y_2 \cdots y_m$ ;
- $r_0 = q_0$ ;
- for  $i$  in  $0 \dots m - 1$ ,  $r_{i+1} \in \delta(r_i, y_i + 1)$ ;
- $r_m \in F$ .

Prove that  $N_{ms}$  accepts a string (i.e. acceptance by Sipser's condition), iff  $N_{mrg}$  accepts the string (i.e. acceptance by the condition given in class).