CpSc 421 | Midterm 2 | **November 15, 2006**

Do problem 0 and any three of 1, 2, 3 and 4.
Graded on a scale of 100 points.

0. (**4 points**) Your name: ___________________________ Your student #: ________________

1. (**32 points**) Consider the two languages described below:

$$\begin{aligned} A_1 &= \mathtt{a}^m\mathtt{b}^n\mathtt{c}^m\mathtt{d}^n \\ A_2 &= \mathtt{a}^m\mathtt{b}^n\mathtt{c}^n\mathtt{d}^m \end{aligned}$$

   where m and n integers that are greater than or equal to 0.

   (a) (**16 points**): One of these languages is a CFL. Which one? Give a CFG for it.

   (b) (**16 points**): One of these languages is not a CFL. Which one? Give a short proof.

2. (**32 points**) Let B be the language: $\{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid \#\mathtt{a}(w) \leq 2\#\mathtt{b}(w)\}$.

   (a) (**24 points**) Give a CFG for B.

   (b) (**8 points**) Give a derivation for the string `abaaabab` using your grammar.

   For partial credit, you can describe a PDA for part (a) and forfeit the points for part (b) (24 points for a correct PDA).

3. (**32 points**) Let Σ^* be a finite alphabet and $C_1, C_2 \subseteq \Sigma^*$ be two languages. Define

$$\begin{aligned} \mathit{shuffle}(C_1, C_2) = \{w \mid {} & \exists x_1, x_2, \ldots, x_k \in \Sigma^*.\ \exists y_1, y_2, \ldots, y_k \in \Sigma^*. \\ & (x_1 \cdot x_2 \cdots x_k \in C_1) \wedge (y_1 \cdot y_2 \cdots y_k \in C_2) \wedge (w = x_1 \cdot y_1 \cdot x_2 \cdot y_2 \cdots x_k \cdot y_k)\} \end{aligned}$$

   Note that this says that the concatenation of *strings* x_1 through x_k produces a string in C_1; the individual x_i might or might not be strings in C_1. Likewise for the strings y_1 through y_k.

   Are the CFLs closed under $\mathit{shuffle}$? Give a short proof with your answer.

4. (**32 points**) Consider the two languages described below:

$$\begin{aligned} D_1 &= \{M \mid \text{for any string } w\text{, if Turing machine } M \text{ accepts } w\text{, it does so in at most 1000 steps}\} \\ D_2 &= \{M \mid \text{for any string } w\text{, if Turing machine } M \text{ accepts } w\text{, it does so in at least 1000 steps}\} \end{aligned}$$

   (a) (**16 points**): One of these languages is Turing decidable. Which one? Give a short justification for your answer.

   (b) (**16 points**): One of these languages is not Turing decidable. Which one? Give a short justification for your answer.

   (c) (**8 points, extra credit**): Is the undecidable language (i.e. D_1 or D_2) Turing recognizable, co-recognizable, both or neither? Give a short justification for your answer.

   You must give a separate justification for each part. In particular, you can't answer (a) and then say that the other language must be undecidable because the problem stated there was one of each.
   Hint: If you're stuck, you can think about how you would solve the problem if "at most (least) 1000 steps" was replaced with "at most (least) 1 step" or "... 2 steps."