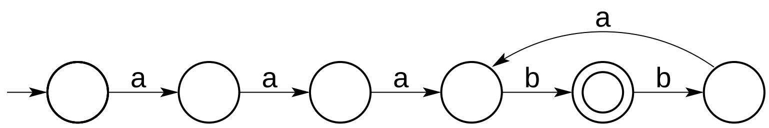
Do problem 0 and any three of problems 1-5.
If you attempt more than three of problems 1-5, please indicate which ones you want graded – otherwise, I'll make an arbitrary choice.

Graded on a scale of 100 points.
You can attempt from 89 to 110 points depending on which problems you choose. If you score over 100, you get to keep the extra credit.

0. (**5 points**) Your name: ________________________________ Your student #: _________________

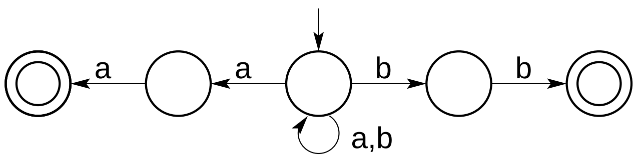
1. (**24 points**)

   (a) (**8** points) Let A_1 be the language recognized by the NFA below:



   Write a regular expression that generates A_1.

   (b) (**8** points) Let A_2 be the language recognized by the NFA below:



   Draw the state diagram for a DFA that recognizes A_2.

   (c) (**8** points) Draw the state diagram for a NFA that recognizes $A_1 \cdot A_2$.

2. (**30 points**) Let $\Sigma = \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$, and let $B = \{w \mid \exists i, j.\ w = \mathsf{a}^i \mathsf{b}^j \mathsf{c}^{i+j}\}$. Prove that B is not regular.

3. (**30 points**) Let C_1 be a language with alphabet $\Sigma = \{\mathsf{a}, \mathsf{b}\}$. Let

$$C_2 \quad = \quad \{w \in \Sigma^* \mid \mathsf{a}^{|w|} \in C_1\}$$

Show that if C_1 is regular, then C_2 is regular as well.
It is sufficient, for example, to describe how to construct an NFA for C_2 given a NFA or DFA (you choose) for C_1. You don't have to give all of the formal details, just describe enough that it is clear that you could write the formulas if you had sufficient time. For example, my solution consists of four English sentences.

4. (**35 points**) Let D_1 be a language with alphabet $\Sigma = \{\mathsf{a}, \mathsf{b}\}$. Let

$$D_2 \quad = \quad \{w \in \Sigma^* \mid \exists x \in D_1.\ x = \mathsf{a}^n \mathsf{b}^n \text{ with } n = |w|\}$$

Show that if D_1 is regular, then D_2 is regular as well.
It is sufficient to describe how to construct an NFA for D_2 given a NFA or DFA for D_1. You don't have to give all of the formal details, just describe enough that it is clear that you could write the formulas if you had sufficient time.

5. (**40 points**) Let $\Sigma = \{\texttt{0}, \texttt{1}\}$, and let E_1 and E_2 be the languages defined below:

$$\begin{aligned} E_1 &= \{w \mid \exists k \in \mathbb{Z}, x \in \Sigma^*.\ (|w| = 10k) \wedge (|x| = k) \wedge (w = x^{10})\} \\ E_2 &= \{w \mid \exists k \in \mathbb{Z}, x \in \Sigma^*.\ (|w| = 10k) \wedge (|x| = 10) \wedge (w = x^{k})\} \end{aligned}$$

One of these languages is regular and the other is not. Identify which language is which.

(a) (**2**0 points) Describe a DFA, NFA or regular expression for the language that is regular – you **don't** need to draw a complete state diagram or write the entire expression; just describe how to construct it.

(b) (**2**0 points) For the other language, prove that it is not regular.