Nondeterministic Finite Automata

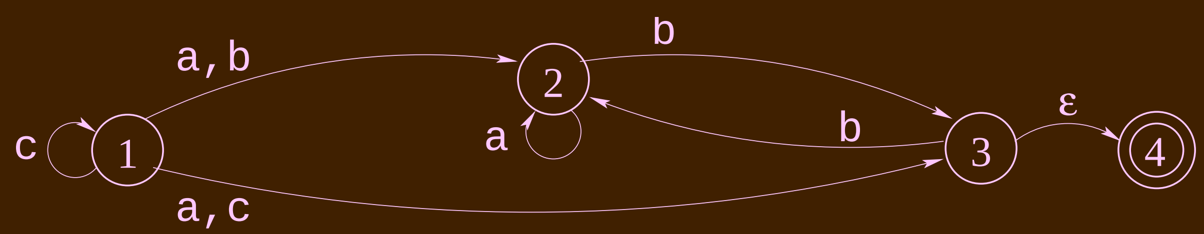
Mark Greenstreet, CpSc 421, Term 1, 2006/07

Lecture Outline

Nondeterministic Finite Automata

- Nondetermistic Finite Automata (NFAs)
- Formal Definition of NFAs
- Applications of Nondeterminism

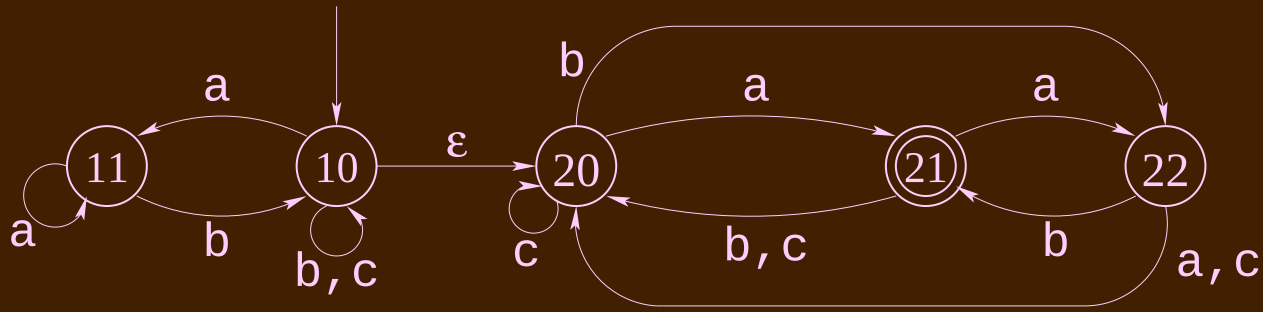
Ingredients of NFAs



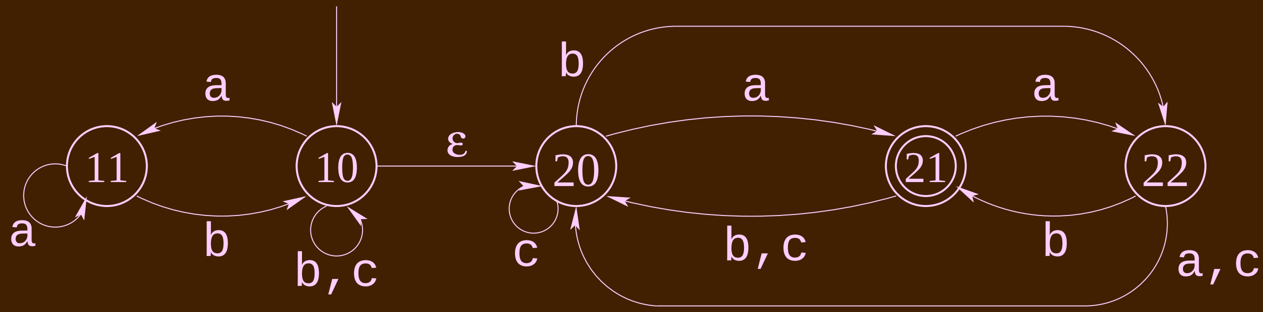
1. A state can have multiple outgoing arcs for the same input symbol.
2. A state can have no outgoing arcs for some input symbol.
3. A state can have arcs that are taken without consuming any input symbol.

- A state may have multiple outgoing arcs labeled with the same input symbol. If that symbol is read, the machine may move along *any* of those arcs.
- A state may have no outgoing arcs labeled for some input symbol. If that symbol is read, the machine immediately rejects.
- A state may have arcs labeled ϵ. When such an arc is taken, no input is read.
- An NFA accepts a string if there is some set of choices for the nondeterministic transitions that lead to an accepting state after reading the complete string.

An Example

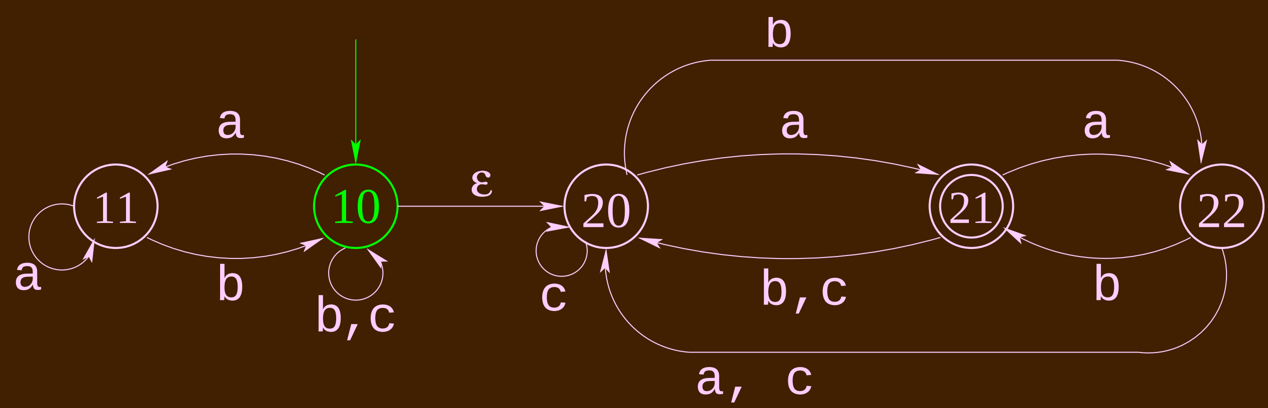


An Example



Consider reading the string: `abcabbbb`.

An Example

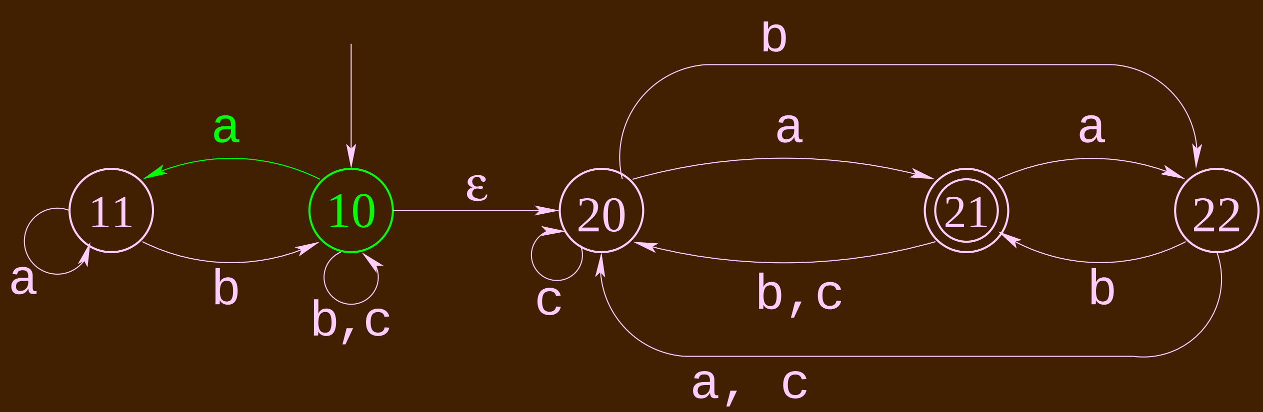


Consider reading the string: abcabbbb.

current state	previously read	current symbol	unread	next state
10	–	–	abcabbbb	

10

An Example

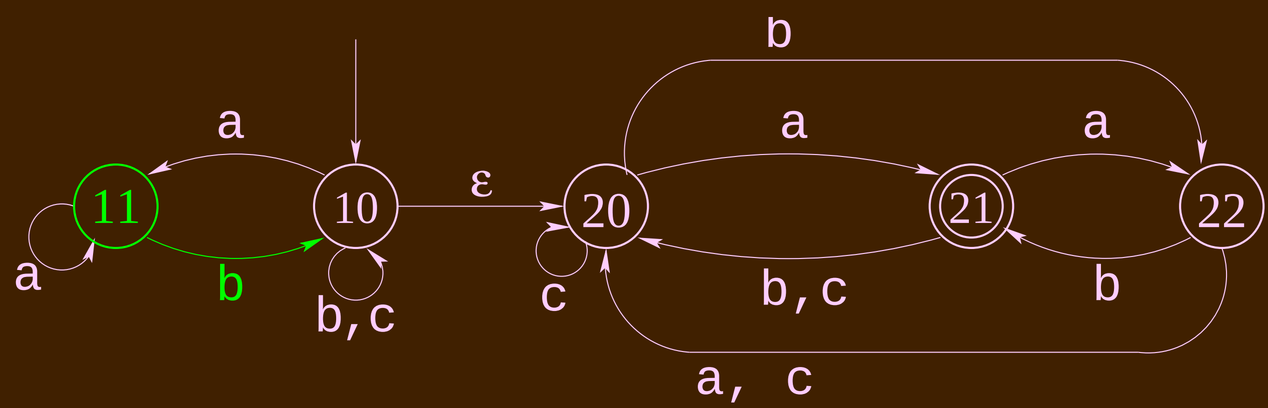


Consider reading the string: abcabbbb.

current state	previously read	current symbol	unread	next state
10	ϵ	a	bcabbbb	11

$10 \xrightarrow{a} 11$

An Example

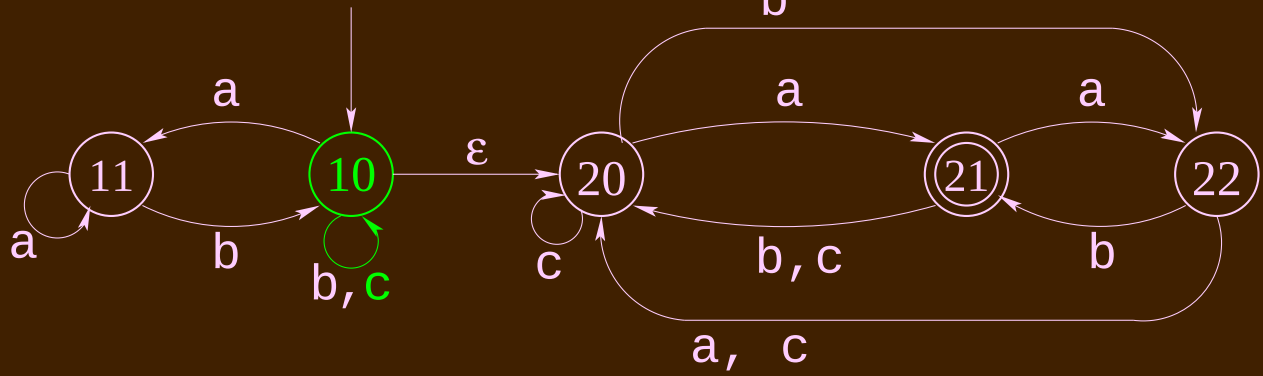


Consider reading the string: abcabbbb.

current state	previously read	current symbol	unread	next state
11	a	b	cabbbb	10

$10 \xrightarrow{a} 11 \xrightarrow{b} 10$

An Example

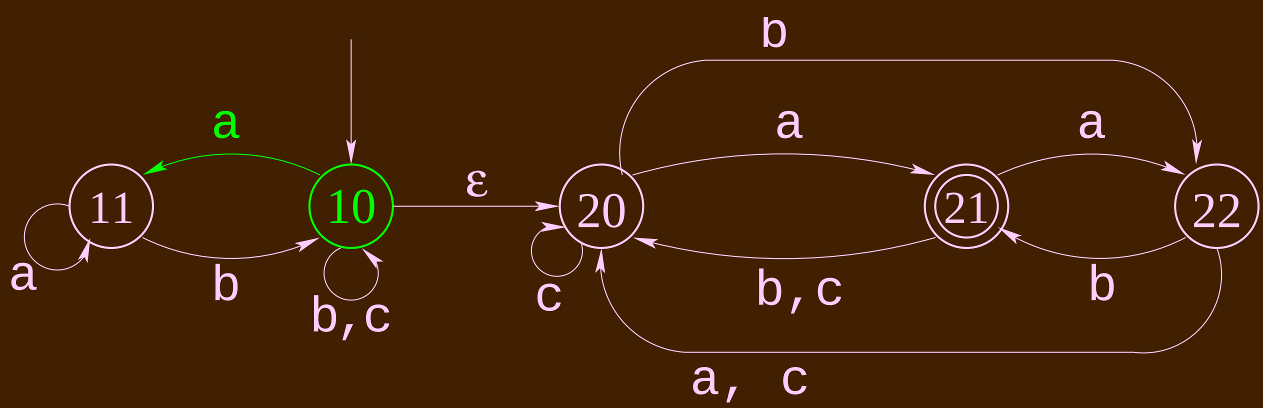


Consider reading the string: abcabbbb.

current state	previously read	current symbol	unread	next state
10	ab	c	abbbb	10

$10 \xrightarrow{a} 11 \xrightarrow{b} 10 \xrightarrow{c} 10$

An Example

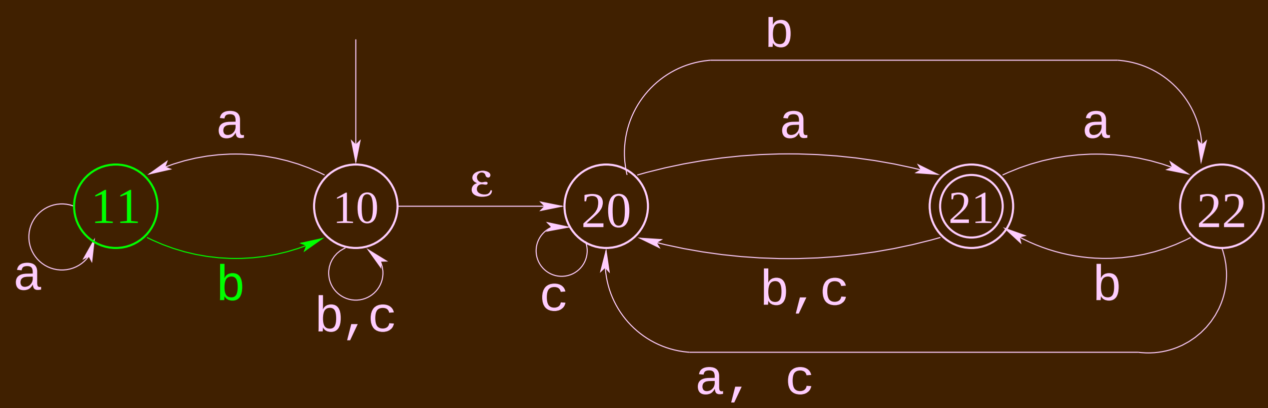


Consider reading the string: abcabbbb.

current state	previously read	current symbol	unread	next state
10	abc	a	bbbb	11

$10 \xrightarrow{a} 11 \xrightarrow{b} 10 \xrightarrow{c} 10 \xrightarrow{a} 11$

An Example

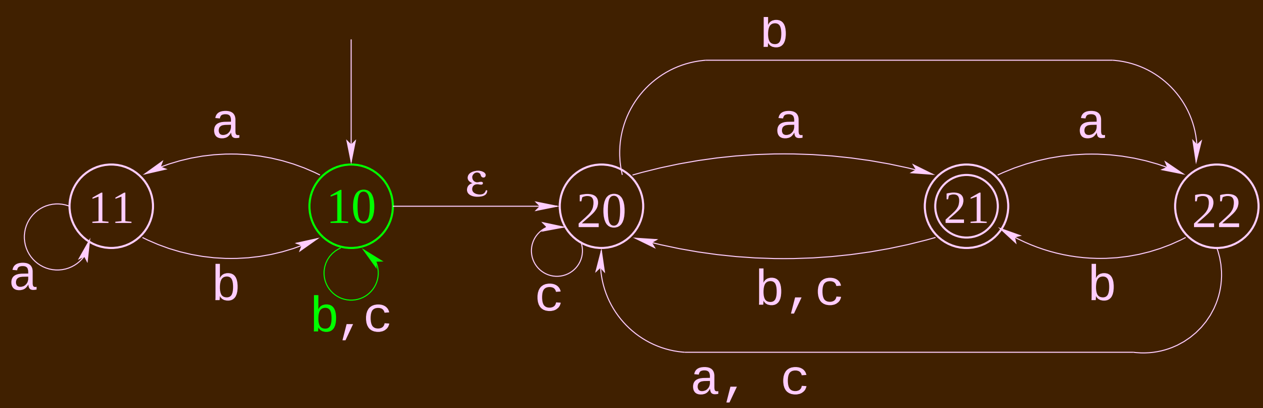


Consider reading the string: abcabbbb.

current state	previously read	current symbol	unread	next state
11	abca	b	bbb	10

$10 \xrightarrow{a} 11 \xrightarrow{b} 10 \xrightarrow{c} 10 \xrightarrow{a} 11 \xrightarrow{b} 10$

An Example

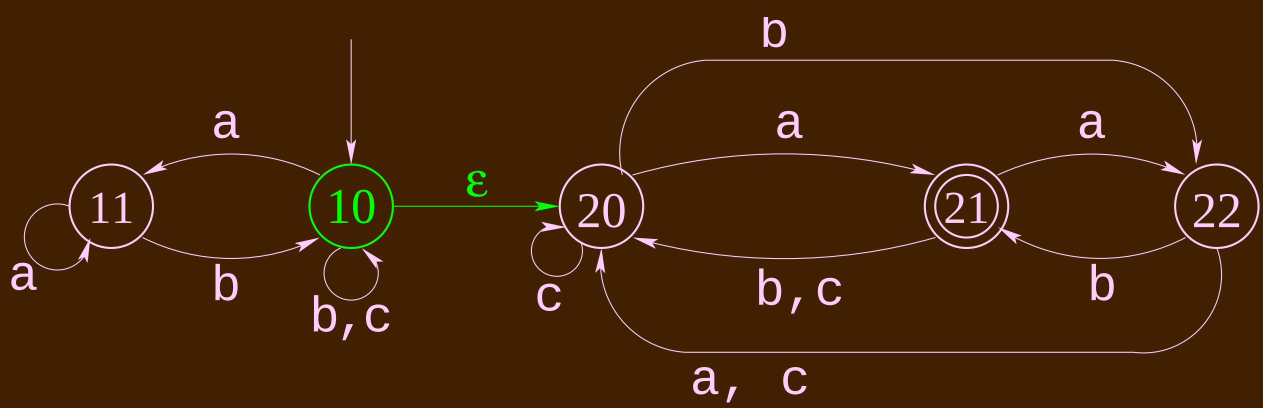


Consider reading the string: abcabbbb.

current state	previously read	current symbol	unread	next state
10	abcab	b	bb	10

$10 \xrightarrow{a} 11 \xrightarrow{b} 10 \xrightarrow{c} 10 \xrightarrow{a} 11 \xrightarrow{b} 10 \xrightarrow{b} 10$

An Example

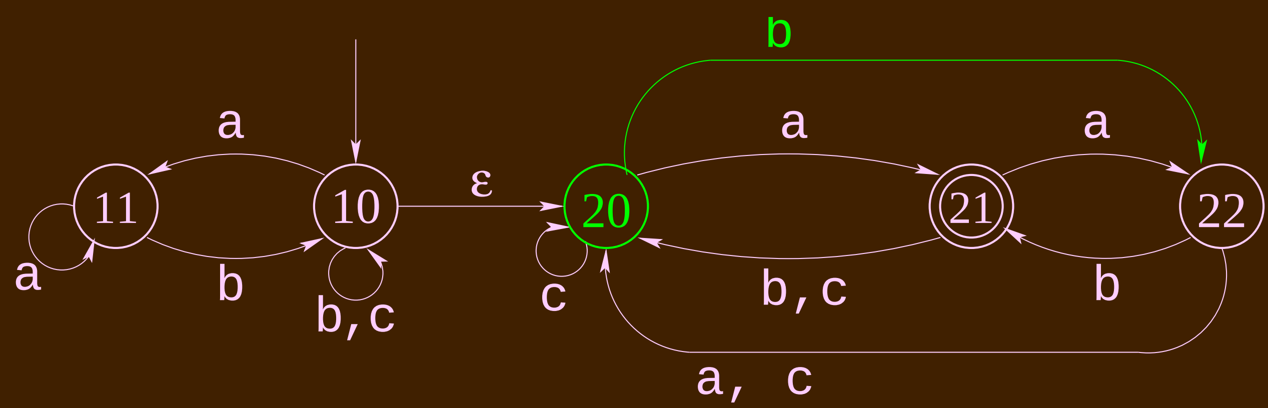


Consider reading the string: abcabbϵbb.

current state	previously read	current symbol	unread	next state
10	abcab	ϵ	bb	20

$10 \xrightarrow{a} 11 \xrightarrow{b} 10 \xrightarrow{c} 10 \xrightarrow{a} 11 \xrightarrow{b} 10 \xrightarrow{b} 10 \xrightarrow{\epsilon} 20$

An Example

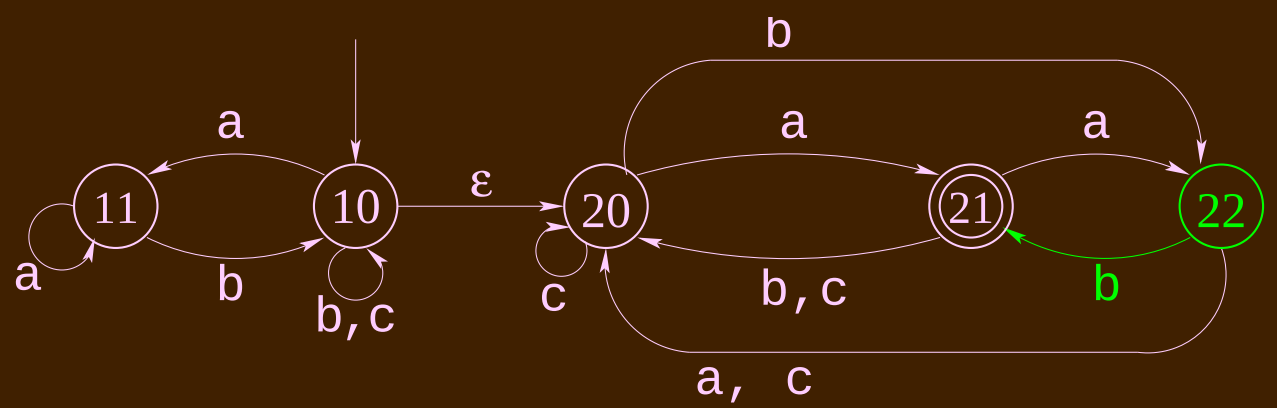


Consider reading the string: abcabbbb.

current state	previously read	current symbol	unread	next state
20	abcabb	b	b	21

$10 \xrightarrow{a} 11 \xrightarrow{b} 10 \xrightarrow{c} 10 \xrightarrow{a} 11 \xrightarrow{b} 10 \xrightarrow{b} 10 \xrightarrow{\epsilon} 20 \xrightarrow{b} 22$

An Example

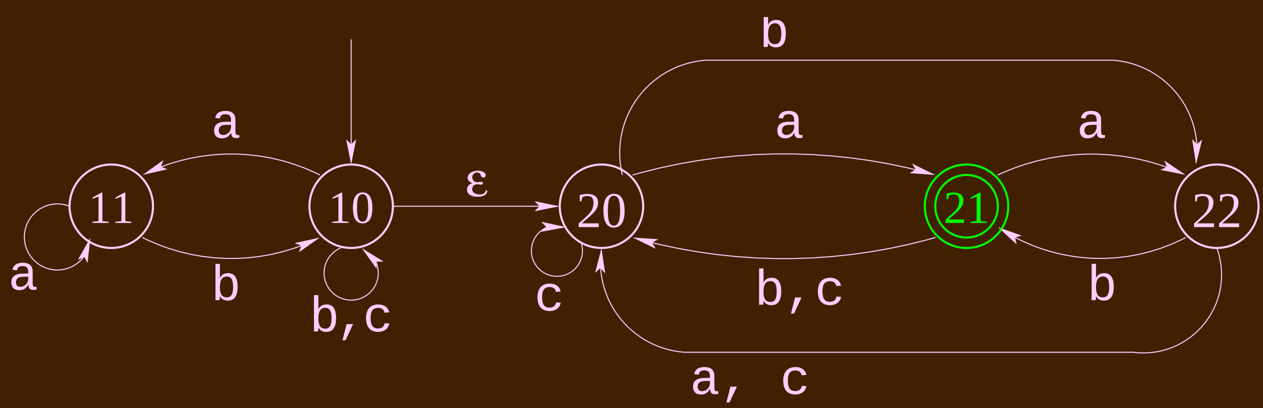


Consider reading the string: abcabbbbb.

current state	previously read	current symbol	unread	next state
21	abcabbb	b	ϵ	22

$10 \xrightarrow{a} 11 \xrightarrow{b} 10 \xrightarrow{c} 10 \xrightarrow{a} 11 \xrightarrow{b} 10 \xrightarrow{b} 10 \xrightarrow{\epsilon} 20 \xrightarrow{b} 22 \xrightarrow{b} 21$

An Example



Consider reading the string: abcabbbb.

current state	previously read	current symbol	unread	next state
22	abcabbbb	–	–	

$10 \xrightarrow{a} 11 \xrightarrow{b} 10 \xrightarrow{c} 10 \xrightarrow{a} 11 \xrightarrow{b} 10 \xrightarrow{b} 10 \xrightarrow{\epsilon} 20 \xrightarrow{b} 22 \xrightarrow{b} 21$ ACCEPTS

Putting it All Together

- Let $\Sigma = \{0, 1\}$.
- Let $S \subseteq \Sigma^*$, such that w is in S iff
  - $w = \epsilon$; or
  - There is a string x in S such that $w = 0x1$ or $w = 1x0$; or
  - There are strings x and y in S such that $w = xy$.
- Prove that w is in S iff the number of O's in w is equal to the number of 1's.
- We'll work this out on the whiteboard.

[Outline section III]