Nondeterminism

Mark Greenstreet, CpSc 421, Term 1, 2006/07

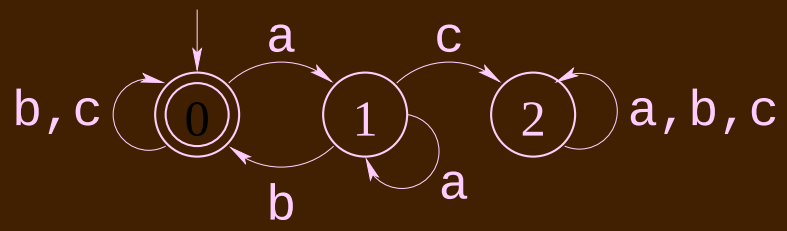
Lecture Outline

Nondeterminism:

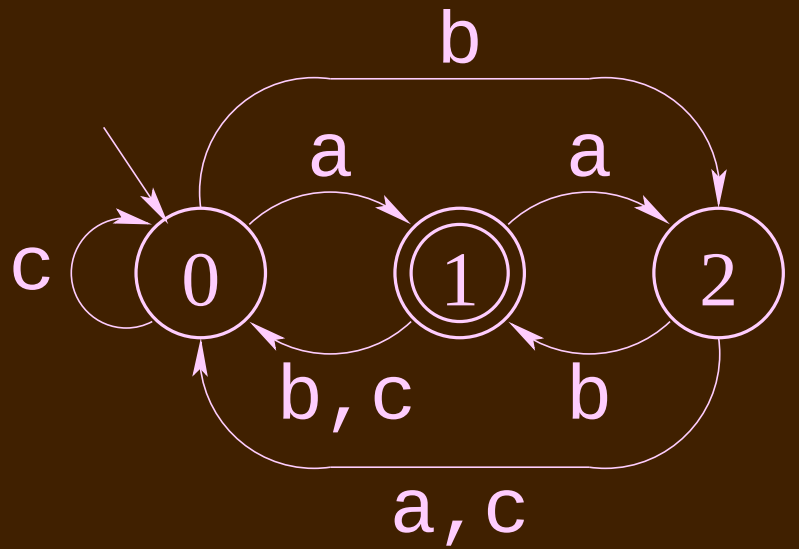
- Closure Properties
- Two-Tape Finite Automata
- Nondeterministic Finite Automata (NFAs)

Our Example Machines

$\Sigma = \{ \mathsf{a}, \mathsf{b}, \mathsf{c} \}$



Finite Automaton M_1



Finite Automaton M_2

$\mathsf{a} \notin L_1, \quad \mathsf{ab} \in L_1, \quad \mathsf{bb} \in L_1, \quad \mathsf{ac} \notin L_1,$

$\mathsf{a} \in L_2, \quad \mathsf{ab} \notin L_2, \quad \mathsf{bb} \in L_2, \quad \mathsf{ac} \notin L_2.$

Closure Properties

- The regular languages are closed under union: $L_1 \cup L_2$ is regular.
- The regular languages are closed under concatenation: $L_1 \circ L_2$ is regular.
- The regular languages are closed under Kleene star (aka “asteration”):

Closure Properties

- The regular languages are closed under union: $L_1 \cup L_2$ is regular.
  - $(w \in L_1 \cup L_2) \Leftrightarrow ((w \in L_1) \vee (w \in L_2))$.
  - $\mathsf{a}, \mathsf{ab}, \mathsf{bb} \in L_1 \cup L_2$.
  - $\mathsf{ac} \notin L_1 \cup L_2$.
- The regular languages are closed under concatenation: $L_1 \circ L_2$ is regular.
- The regular languages are closed under Kleene star (aka “asteration”):

Closure Properties

- The regular languages are closed under union: $L_1 \cup L_2$ is regular.
- The regular languages are closed under concatenation: $L_1 \circ L_2$ is regular.
  - $(w \in L_1 \circ L_2) \Leftrightarrow (\exists x \in L_1.\, \exists y \in L_2.\, w = xy)$
  - $\mathsf{a}, \mathsf{bb}, \mathsf{abba}, \mathsf{ababcbabbabbabba} \in L_1 \circ L_2$.
  - $\mathsf{ab}, \mathsf{b}, \mathsf{ac}, \mathsf{abc} \notin L_1 \circ L_2$.
- The regular languages are closed under Kleene star (aka “asteration”):

Closure Properties

- The regular languages are closed under union: $L_1 \cup L_2$ is regular.
- The regular languages are closed under concatenation: $L_1 \circ L_2$ is regular.
- The regular languages are closed under Kleene star (aka “asteration”):
  - $(w \in L^*) \Leftrightarrow \exists m \in \mathbb{N}.\ w \in L^m$.
  - $L^0 = \{\epsilon\}$, $L^1 = L$, $L^2 = L \circ L$.
  - $L^{k+1} = L^k \circ L$.
  - .