Regular Languages

Mark Greenstreet, CpSc 421, Term 1, 2006/07

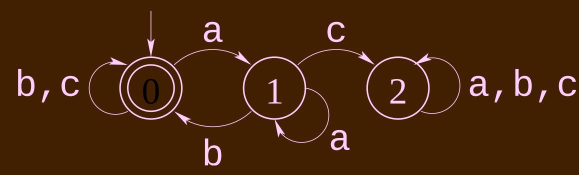
Lecture Outline

Regular Languages

- Finite Automata and Regular Languages
- Closure Properties

A Finite Automaton

$\Sigma = \{ a, b, c \}$



- Processing abcaabc:

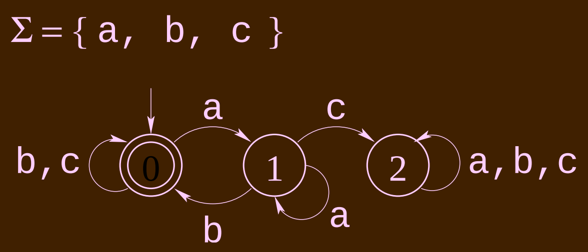
previously processed input	current input symbol	pending input	current state	next state
ϵ	a	bcaabc	0	1
a	b	caabc	1	0
ab	c	aabc	0	0
abc	a	abc	0	1
abca	a	bc	1	1
abcaa	b	c	1	0
abcaab	c	ϵ	0	0
abcaabc	–	ϵ	0	

The string is accepted. ☺

Formal Definition of Finite Automata

- A Finite Automaton is a 5-tuple, $(Q, \Sigma, \delta, q_0, F)$ where
  - Q is a finite set of states.
  - Σ is the input alphabet, a finite set of symbols.
  - $\delta : Q \times \Sigma \to Q$ is the state transition function.
  - $q_0 \in Q$ is the initial state.
  - $F \subseteq Q$ is the set of accepting states.

Our Example Machine Again



- $Q = \{0, 1, 2\}$.
- $\Sigma = \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$.
- $\delta(q, c)$, with q ranging over the rows and C spanning the columns a, b, c:

$\delta(q,c)$	a	b	c
0	1	0	0
1	1	0	2
2	2	2	2

- $q_0 = 0$.
- $F = \{0\}$.

Extending δ to strings

- $\delta(q, \epsilon) = q$.
- $\delta(q, x \cdot c) = \delta(\delta(q, x), c)$.
- Note that the definition of δ parallels the inductive definition of strings from the Sept. 8 lecture where we said that a string is either
  - ϵ, or
  - $x \cdot c$, where x is a string and $c \in \Sigma$.
- Let $M = (Q, \Sigma, \delta, q_0, F)$.
  M accepts w iff $\delta(q_0, w) \in F$.

Our example, again

Processing the string abcaabc.

$$\begin{aligned}
&\delta(q_0, \mathsf{abcaabc}) \\
&= \delta(0, \mathsf{abcaabc}), \qquad q_0 = 0 \\
&= \delta(\delta(0, \mathsf{abcaab}), \mathsf{c}) \\
&= \delta(\delta(\delta(0, \mathsf{abcaa}), \mathsf{b}), \mathsf{c}) \\
&= \ldots \\
&= \delta(\delta(\delta(\delta(\delta(\delta(\delta(\delta(0, \epsilon), \mathsf{a}), \mathsf{b}), \mathsf{c}), \mathsf{a}), \mathsf{a}), \mathsf{b}), \mathsf{c}),
\end{aligned}$$

now we can s

Our example, continued

$$\begin{aligned}
&\delta(q_0, \mathsf{abcaabc}) \\
&= \delta(\delta(\delta(\delta(\delta(\delta(\delta(\delta(0,\epsilon),\mathsf{a}),\mathsf{b}),\mathsf{c}),\mathsf{a}),\mathsf{a}),\mathsf{b}),\mathsf{c}), \quad \text{now we can s} \\
&= \delta(\delta(\delta(\delta(\delta(\delta(\delta(0,\mathsf{a}),\mathsf{b}),\mathsf{c}),\mathsf{a}),\mathsf{a}),\mathsf{b}),\mathsf{c}) \\
&= \delta(\delta(\delta(\delta(\delta(\delta(1,\mathsf{b}),\mathsf{c}),\mathsf{a}),\mathsf{a}),\mathsf{b}),\mathsf{c}) \\
&= \ldots \\
&= \delta(0,\mathsf{c}) \\
&= 0
\end{aligned}$$

Thus, $\delta(q_0, \mathsf{abcaabc}) \in F$, and the string is accepted.