CpSc 421 Homework 9 **Due: Nov. 20, 4pm**

Extra Credit

Note: All problems on this homework set are extra-credit. You may turn in solutions for up to four of the problems below. Turning in a solution for any part of a problem counts as attempting the entire problem.
Have fun!

1. (**20 points**) Let A be a CFL and B be regular. Prove that $A \cap B$ is a CFL.
2. (**30 points**) Let A be the language $\{x \mid \exists w.\ x = ww\}$. We showed in class that A is not a CFL. Prove that $\overline{A}$ *is* a CFL.
3. (**30 points**) A DPDA is a deterministic, pushdown automaton. Formally, a DPDA, $D = (Q, \Sigma, \Gamma, \delta, q_0, F)$. From any configuration, a DPDA has exactly one possible move. We need to allow ϵ moves so that in appropriate situations the DPDA can push more symbols onto the stack than it pops off. If the DPDA has an ϵ move possible, then there must only be one such move, and no other move may be possible. Finally, the input alphabet includes a special symbol, $\dashv$. This is an endmarker – the last symbol of any input string is $\dashv$ and $\dashv$ may not appear before the last symbol of the string.

   We can describe DPDAs by drawing transition diagrams just as we did for (non-deterministic) PDAs. For a DPDA, there may not be multiple arrows out of a state that could be taken for the same input symbol and stack symbol.

   (a) (**10 points**) Draw the transition diagram for a DPDA that accepts
   $$\{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid \#\mathtt{a}(w) < \#b(w)\}$$
   (b) (**10 points**) Prove that the class of languages accepted by DPDAs is closed under complement.
   (c) (**10 points**) Prove that the class of languages accepted by DPDAs is not closed under union.
4. (**20 points**) Let $A_c = \{M\#w \mid \text{ TM } M \text{ writes symbol } c \text{ on its tape when run with input } w\}$. Show that the language A_c is Turing undecidable.
5. (**30 points**) Let $\mathit{NoWriteOverInput}$ be the class of Turing machines that may not alter their input strings. If $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$ is a Turing machine, then $M \in \mathit{NoWriteOverInput}$ iff for every $c \in \Sigma$ and $q \in Q$:
   $$\delta(q, c) \quad = \quad (q', c, d)$$
   for some $q' \in Q$ and some $d \in \{L, R\}$ (note that M writes the same symbol that it read).

   (a) (**15 points**) Let M be a Turing machine in $\mathit{NoWriteOverInput}$. Prove that $L(M)$ is regular.
   (b) (**15 points**) Prove that the language $\{M\#w \mid M \text{ accepts } w\}$ is undecidable even if we restrict M to be in $\mathit{NoWriteOverInput}$.
6. I hope to add a few more problems in the next day or two. I'll announce it when I add them.