CpSc 421 Homework 8 **Due: Nov. 10**
No Late Homework Accepted

1. (**15 points**) Let $A_1 = \{M\#w \mid M$ halts after at most $|w|^{|w|}$ steps when run with input $w\}$. Show that language A is Turing decidable.

2. (**30 points**) Let $A_2 = \{M \mid \exists w.\ M\#w \in A_1\}$.

   (a) (**15 points**) Show that language A_2 is not Turing decidable.

   (b) (**15 points**) Show that language A_2 is Turing recognizable.

3. (**40 points**)

   (a) (**10 points**) Show that the class of Turing decidable languages is closed under complement.

   (b) (**10 points**) Show that the class of Turing decidable languages is closed under star.

   (c) (**10 points**) Show that the class of Turing recognizable languages is not closed under complement.

   (d) (**10 points**) Show that the class of Turing recognizable languages is closed under star.

4. (**30 points**) A *linear bounded automaton* (LBA) is a Turing Machine with a bounded tape; it cannot move its head past *either* end of the input string. For example, you can assume that the input string has the form $\vdash v \dashv$ where $\vdash$ is a special left endmarker (that appears nowhere in v) and $\dashv$ is a special right endmarker (that appears nowhere in v). All transitions from $\vdash$ preserve the $\vdash$ and move the head to the right. All transitions from $\dashv$ preserve the $\dashv$ and move the head to the left. Let

$$A_{LBA} \quad = \quad \{M\#w \mid M \text{ is an LBA that accepts } w\}.$$

   A_{LBA} is Turing decidable (see Sipser Lemma 5.8). Thus, the halting problem for LBA's is Turing decidable as well.

   Prove that there is some language, B such that B is Turing decidable but B is not accepted by any LBA. (**Hint:** use diagonalization.)