1. (**90 points**) For each language below, determine whether or not the language is context-free, and justify your answer. For example, if the language **is** context-free, you can give a CFG or PDA for the language. If it is not context-free, you can use the pumping lemma for CFLs to prove this. You may use relationships to other languages that we have shown to be context-free or not, and you can use the results from previous parts of this problem when solving subsequent parts.

   (a) **(15 points)** $A = \mathtt{a}^n\mathtt{b}^{(n^2)}$.

   (b) **(15 points)** $B = \mathtt{a}^n\mathtt{b}^n c^*$.

   (c) **(15 points)** $C = \mathtt{a}^n\mathtt{b}^n\mathtt{c}^*\mathtt{a}^*\mathtt{b}^m\mathtt{c}^m$.

   (d) **(15 points)** $D = \mathtt{a}^i\mathtt{b}^j,\ i \neq j$.

   (e) **(15 points)** $E = \overline{E_0}$, where $E_0 = \mathtt{a}^n\mathtt{b}^n\mathtt{c}^n$, and the alphabet is $\{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$.

   (f) **(15 points)** $F = \{w \in \{0,1\}^* \mid (w = w^{\mathcal{R}}) \wedge (\#0(w) = \#1(w))\}$

2. (**20 points**) Based upon some of the answers to question 1 and languages that we have shown in class (and in Sipser) to be CFLs or not, you can conclude that the CFLs are neither closed under intersection nor complement. Explain.

3. **(20 points)** Consider the following attempt to show that $\mathtt{0}^n\mathtt{1}^n\mathtt{2}^n$ is context-free.

   It's easy to make a PDA check that its input is of the form $\mathtt{0}^*\mathtt{1}^*\mathtt{2}^*$. Let P be such a PDA that takes the following additional actions as it reads its input (see Figure 1):

   - The stack alphabet for p is $\{\$, \Diamond\}$. Initially, P pushes a \$ onto its stack.
   - Each time P reads a 0, it pushes two $\Diamond$ symbols onto its stack.
   - Every other time that P reads a 1, it pops a $\Diamond$ off of its stack.
   - Every other time that P reads a 2, it pops another $\Diamond$ off of its stack.
   - After reading n 0's, P has pushed $2n$ $\Diamond$'s onto its stack. If P reaches a configuration after reading the entire input where the endmarker, \$, is on the top of the stack, then P must have popped n $\Diamond$'s off its stack while reading 1's and another n while reading 2's. This means that the string is in $\mathtt{0}^n\mathtt{1}^n\mathtt{2}^n$; P pops the \$ off of the stack and accepts.

   We've shown that $\mathtt{0}^n\mathtt{1}^n\mathtt{2}^n$ is recognized by a PDA. Therefore, it is context free.

   Explain what is wrong with this "proof"

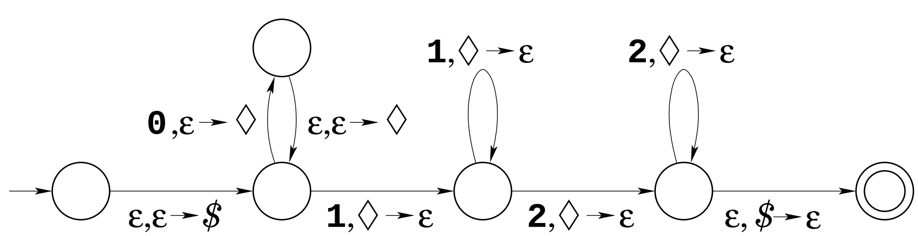

Figure 1: A PDA that alledgedly accepts $\mathtt{0}^n\mathtt{1}^n\mathtt{2}^n$