1. **(25 points)** Sipser, problem 1.31.
For any string $w = w_1 w_2 \ldots w_n$, the **reverse** of w, written $w^{\mathcal{R}}$, is the string w in reverse order, $w^{\mathcal{R}} = w_n \ldots w_2 w_1$. For any language A, let

$$A^{\mathcal{R}} \quad = \quad \{w^{\mathcal{R}} \mid w \in A\}.$$

Prove that if A is regular, then $A^{\mathcal{R}}$ is regular as well.

2. **(25 points)** Sipser, problem 1.32.
Let

$$\Sigma_3 \quad = \quad \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \cdots \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

Σ_3 contains all size 3 columns of 0s and 1s. A string of symbols in Σ_3 gives three rows of 0s and 1s. Consider each row to be a binary number with the most significant bit first. For example, let

$$w \quad = \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

The first row of w is the binary representation of 7, the second row corresponds to 5, and the third row corresponds to 12.

Let

$$B \quad = \quad \{w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top two rows}\}.$$

Show that B is regular. (Hint: Working with $B^{\mathcal{R}}$ is easier. You can use the result that you were asked to prove for question 1).

3. **(25 points)** Let u and v be strings with $|u| = |v|$. We define $\mathit{weave}(u, v)$ as shown below:

$$\begin{array}{rcl} \mathit{weave}(\epsilon, \epsilon) & = & \epsilon \\ \mathit{weave}(x \cdot a, y \cdot b) & = & \mathit{weave}(x, y) \cdot a \cdot b \end{array}$$

For example, $\mathit{weave}(\texttt{cat}, \texttt{dog}) = \texttt{cdaotg}$ and $\mathit{weave}(\texttt{srn}, \texttt{tig}) = \texttt{string}$.

For any language A, let

$$\mathit{half}(A) \quad = \quad \{u \mid \exists v \in \Sigma^{|u|}.\ \mathit{weave}(u, v) \in A\}$$

Prove that if A is regular, then $\mathit{half}(A)$ is regular as well.

4. **(10 points)** (from Sipser 1.20).
For each of the following languages, give two strings that are members and two strings that are *not* members – a total of four strings for each part. Assume that the alphabet $\Sigma = \{\texttt{a}, \texttt{b}\}$ in all parts.

   (a) $\texttt{a}^*\texttt{b}^*$.

   (b) $\texttt{a}^*(ba)^*\texttt{b}^*$.

   (c) $\texttt{a}^* \cup \texttt{b}^*$.

   (d) $(\texttt{aaa})^*$.

   (h) $(\texttt{a} \cup \texttt{ba} \cup \texttt{bb})\Sigma^*$.

5. **(15 points)** (from Sipser 1.22).
In certain programming languages, comments appear between delimiters such as `/#` and `/#`. (We're using `/#` instead of `/*` as for comments in C to avoid confusion of the character $\star$ with the regular expression operator, *.) Let C be the language of all valid delimited comment strings. A member of C must begin with `/#` and end with `#/` but have no intervening `#/`. For simplicity, we'll say that comments themselves are written with only the symbols `a`, `b`; hence the alphabet of C is $\Sigma = \{\texttt{a}, \texttt{b}, /, \#\}$.

   (a) Give a DFA that recognized C.

   (b) Give a regular expression that generates C.

   Note: As described by Sipser, the text of the comment cannot contain the symbols / or #. Thus, `/#abbabaab#/` is a valid comment but `/#ab/babb#ba##/` is not a valid comment. You may make this assumption in your solution – it makes the solution easier.