1. (Sipser problem 5.21, **20 points**) Let $AMBIG_{CFG} = \{[G] \mid G \text{ is an ambiguis CFG}\}$ (where $[G]$ denotes a string that represents the grammar). Show that $AMBIG_{CFG}$ is undecidable.
   Hint: You can use a reduction from PCP. Given an instance

$$P = \left\{ \boxed{\begin{array}{c} t_1 \\ \hline b_1 \end{array}}, \boxed{\begin{array}{c} t_2 \\ \hline b_2 \end{array}}, \ldots \boxed{\begin{array}{c} t_k \\ \hline b_k \end{array}} \right\},$$

   of the Post Correspondence Problem, construct a CFG G with the rules

$$\begin{array}{lcl} S & \rightarrow & T \mid B \\ T & \rightarrow & t_1 T \mathsf{a}_1 \mid \ldots \mid t_k T \mathsf{a}_k \mid t_1 \mathsf{a}_1 \mid \ldots \mid t_k \mathsf{a}_k \\ B & \rightarrow & b_1 B \mathsf{a}_1 \mid \ldots \mid b_k B \mathsf{a}_k \mid b_1 \mathsf{a}_1 \mid \ldots \mid b_k \mathsf{a}_k \ , \end{array}$$

   where $\mathsf{a}_1, \ldots, \mathsf{a}_k$ are new terminal symbols. Prove that this reduction works.

2. (Sipser problems 5.22 and 5.23, **20 points**)
   (a) Show that A is Turing-recognizable iff $A \leq_m A_{TM}$.
   (b) Show that A is Turing-decidable iff $A \leq_m 0^*1^*$.

3. (Sipser problem 5.24, **20 points**) Let $J = \{w \mid \text{either } w = 0x \text{ for some } x \in A_{TM} \text{ or } w = 1y \text{ for some } y \notin A_{TM}\}$. Show that neither J nor $\overline{J}$ is Turing-recognizable.

4. (Sipser problem 5.34, **30 points**) Consider the problem of determing whether a PDA accepts some string of the form $\{ww \mid w \in \{0,1\}^*\}$. Use the computation history method to show that this problem is undecidable.