1. **(20 points)** Recall the inductive definition for the set, S, of all strings in $\{0,1\}^*$ from the September 8 lecture notes: w is in S iff
   - $w = \epsilon$; or
   - There is a string x in S such that $w = 0x1$ or $w = 1x0$; or
   - There are strings x and y in S such that $w = xy$.

   (a) **(10 points)** Give an inductive definition for a set, T, that contains all strings that have more 1's than 0's.

   (b) **(10 points)** Give a proof that your solution to part (a) is correct.

   Hint: You may find it helpful to use S in your definition of T.

2. **(20 points)** Let $\Sigma = \{0,1,2\}$. Let $\subseteq \Sigma^* H$ be the language that contains a string w iff
   - $w = \epsilon$; or
   - There are strings x and y in H such that $w \in \{0x1y2, 0x2y1, 1x0y2, 1x2y0, 2x0y1, 2x1y0\}$.

   (a) **(10 points)** Prove that for each string, w in H, the number of 0's, 1's and 2's in w are all equal to each other.

   (b) **(10 points)** Does H contain all strings that have an equal number of 0's, 1's and 2's? Give a short proof for your answer.

3. **(30 points)** Let $\Sigma = \{\mathsf{a}, \mathsf{b}\}$. Figure 1 depicts three finite state machines that read inputs from this alphabet. Let L_a, L_b, and L_c be the languages accepted by DFA (a), DFA (b), and DFA (c) respectively.

   (a) **(9 points)** For each of L_a, L_b, and L_c, list three strings in Σ^* that are in the language and three strings in Σ^* that are not in the language.

   (b) **(12 points)** Write a short description of each of the language, L_a, L_b and L_c.

   (c) **(9 points)**
   Is $L_a = L_b$, $L_a \subset L_b$, $L_a \supset L_b$, or none of these?
   Is $L_b = L_c$, $L_b \subset L_c$, $L_b \supset L_c$, or none of these?
   Is $L_a = L_c$, $L_a \subset L_c$, $L_a \supset L_c$, or none of these?
   Give a short justification of your answers.

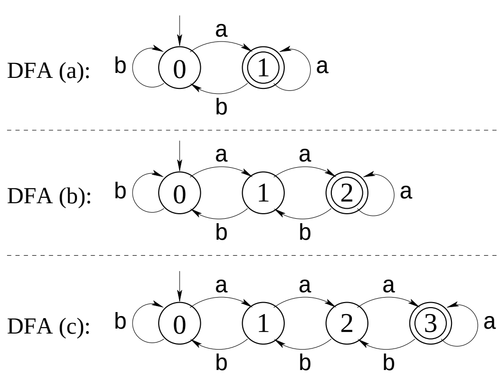

Figure 1: Finite state machines for question 3