Today's lecture: The Pumping Lemma for CFLs

Announcements:

HW 2 due October 14.
I haven't posted solutions to HW1 yet. I want to make sure that you have several days between seeing the solutions to HW1 and the due date for HW2.

One more office hour:
Tuesdays, 8-9am, my office (ICICS 323).
I'll try it for for the next three weeks (i.e. until the midterm). If people show up, I'll continue it.

Reading:

Today: Non-Context-Free Languages
Read: *Kozen* lecture 22 or *Sipser* 2.3.

October 14: Non-deterministic, Pushdown Automata
Read: *Kozen* lecture 23 or *Sipser* 2.2.

October 17: From CFLs to PDAs
Read: *Kozen* lecture 24 (or *Sipser* 2.2).

October 19: From PDAs to CFLs
Read: *Kozen* lecture 25 (or *Sipser* 2.2).

October 21: Deterministic PDAs
Read: *Kozen* lectures E and F.

October 24: Parsing
Read: *Kozen* lecture 26 (not in Sipser)

October 26: Midterm: In class.

October 28: A Parsing Algorithm
Read: *Kozen* lecture 27 (not in Sipser)

October 31: LALR Parsing
Read: TBD

I. Binary trees

A. How many leaves can a binary tree of depth d have?

1. At least $d+1$

Proof by induction on d.

2. At most 2^d

Proof by induction on d.

B. What is the depth of a tree with n leaves?

1. At least $\lceil \log_2(n) \rceil$

2. At most $n-1$

3. Both claims follow from the corresponding bounds for number of leaves versus depth.

C. Application to CFLs

1. Let A be a CFL.

2. Let (N, Σ, P, S) be a CFG for A in Chomsky Normal Form (CNF).

3. A parse-tree for a CNF grammar is a binary tree, with degree-1 nodes just above the leaves.

a. A parse tree with n non-terminals has a depth of at least $\lceil \log_2(n) \rceil$.

b. Let $k = |N|$.

c. Consider a parse tree with 2^{k+1} non-terminals:

i. This tree must have at least one non-terminal at depth $k+2$.

ii. There are at least $k+1$ non-terminals on the path from the start-symbol to this terminal.

iii. At least one non-terminal is repeated.

have

d. This lets us "pump" the string.

II. The pumping lemma for CFLs

A. Statement.

B. Proof.

C. Games with a demon.

III. An example: $x | \exists w.\ x = ww$ is not a CFL

A. Proof.

B. $\{x |\ (\exists w.\ x = ww)\}$ is a CFL.

C. Conclusion: CFLs are not closed under complement.