CpSc 421 Homework 7 **Due: Dec. 14**

- No late homework accepted.
- This is an optional homework assignment. I will compute your final grade by taking the highest six scores from the seven homework assignments.
- Do any five of the six problems below. Do not turn-in solutions for more than five problems.

1. **(20 points):** Kozen, Homework 9, problem 4.
   Prove that an r.e. set is recursive iff there exists an enumeration machine that enumerates it in increasing order.
2. **(20 points):** Kozen, Miscellaneous exercise, problem 111.
   One of the following sets is r.e. and the other is not. Which is which? Give proof for both.
   (a) $\{M \mid L(M)$ contains at least 481 elements$\}$
   (b) $\{M \mid L(M)$ contains at most 481 elements$\}$
3. **(20 points):** Kozen, Miscellaneous exercises, problem 37, parts a, b, i and j.
   Which of the following sets are regular and which are not? Give justification.
   (a) $\{a^n b^{2m} \mid n \geq 0$ and $m \geq 0\}$
   (b) $\{a^n b^m \mid n = 2m\}$
   (c) $\{a^n b^m \mid n \geq m$ and $m \leq 481\}$
   (d) $\{a^n b^m \mid n \geq m$ and $m \geq 481\}$
4. **(20 points):** Kozen, Miscellaneous exercises, problem 76.
   Consider the set
   $$a^* b^* c^* - \{a^n b^n c^n \mid n \geq 0\}$$
   the set of all strings of a's followed by b's followed by c's such that the number of a's, b's and c's are not all equal.
   (a) Give a CFG for the set, and prove that your grammar is correct.
   (b) Give an equivalent PDA.
5. **(20 points):** Kozen, Miscellaneous exercises, problem 106.
   Is it decidable, given $M\#y$, whether the Turing machine M ever writes a nonblank symbol on its tape when run with input y? Why or why not?
6. **(20 points):** Kozen, Miscellaneous exercises, problem 108.
   Tell whether the following problems are decidable or undecidable. Give proof.
   (a) Given a TM M and a string Y, does M every write the symbol $\#$ on input y?
   (b) Given a CFG G, does G generate all strings except ϵ?
   (c) Given an LBA M, does M accept a string of even length?
   (d) Give a TM M, are there infinitely many TMs equivalent to M?