1. **(20 points):** Do Kozen, Homework 8, problem 1.
   Describe a TM that accepts the set $\{a^n | n \text{ is a power of } 2\}$. Your description should be at the level of the descriptions in Kozen Lecture 29 of the TM that accepts $\{ww | w \in \Sigma^*\}$ and the TM that implements the sieve of Eratosthenes. In particular, do not give a list of transitions.

2. **(30 points):** Do Kozen, Homework 8, problem 2.
   A *linear bounded automaton* (LBA) is exactly like a one-tape Turing machine, except that the input string $x \in \Sigma^*$ is enclosed in left and right endmarkers, $\vdash$ and $\dashv$, which may not be overwritten, and the machine is constrained to never move left of the $\vdash$ nor right of the $\dashv$. It may read and write all it wants between the endmarkers.

   (a) **(8 points)** Give a rigorous, formal definition of deterministic, linearly bounded automata, including a definition of configurations and acceptance. Your definition should begin as follows: "A *deterministic linearly bounded automaton (LBA)* is a 9-tuple
   $$M = (Q, \Sigma, \Gamma, \vdash, \dashv, \delta, s, t, r),$$
   where Q is a finite set of *states*, ..."

   (b) **(5 points)** Let M be a linear bounded automaton with state set Q of size k and tape alphabet Γ of size m. How many possible configurations are there on input x, $|x| = n$?

   (c) **(7 points)** Argue that the halting problem for deterministic linearly bounded automata is decidable. (*Hint:* you need to be able to detect after a finite time if the machine is in an infinite loop. Presumably the result of part (b) would be useful here.)

   (d) **(10 points)** Prove by diagonalization that there exists a recursive set that is not accepted by any LBA.

3. **(20 points):** I mentioned in class that we can construct a Turing machine that addresses its tape as if it were memory. In this problem, you'll show me how to do it.

   Consider a Turing machine with tape alphabet $\{1, 1', a, a', \square\}$. Start with a configuration where the tape is of the form:
   $$0^{k_0}10^{k_1}10^{k_2}1\ldots0^{k_n}1\square^\omega$$
   We can interpret this tape as storing the sequence of integers k_0, k_1, $\ldots k_n$. Describe a Turing machine that starts in state q_1 and ends in state q_2 after replacing k_n with k_{k_n}. For example, if the tape is
   $$0001011000001000000001010001\square^\omega$$
   when the machine is in state q_1, the machine will do its thing and reach state q_2 with the tape holding
   $$000101100000100000000101000001\square^\omega$$
   You should specify your machine as a labeled state transition diagram like the ones that I've presented in class.

   This operation corresponds to reading from memory.

4. **(30 points):** Do Kozen, Homework 9, problem 2.
   Prove that it is undecidable whether two given Turing machines accept the same set. (This problem is analogous to determining whether two given PASCAL programs are equivalent.)