CpSc 421 Homework 3 **Due: Oct. 21**

NO LATE HOMEWORK ACCEPTED

1. **(30 points):** (Question 2 from Kozen Homework 5)
   Prove that the CFG:
   $$S \rightarrow aSb \mid bSa \mid SS \mid \epsilon$$
   generates the set of all strings of $\{a, b\}$ with equally many a's and b's. (**Hint:** Characterize elements of the set in terms of the graph of the function $\#b(y) - \#a(y)$ as y ranges over prefixes of x, as we did with balanced parentheses.)

2. **(30 points):** (Question 2 from Kozen Homework 6)
   Construct a pushdown automaton that accepts the set of strings in $\{a, b\}^*$ with equally many a's and b's. Specify all transitions.

3. **(40 points):** Let T be the language over the alphabet $\{\mathtt{[}, \mathbf{I}, \mathtt{]}\}$ such that every $\mathtt{[}$ is followed by its matching $\mathbf{I}$, and every $\mathbf{I}$ is followed by its matching $\mathtt{]}$, and the total number of $\mathtt{[}$ symbols and the total number of $\mathtt{]}$ symbols in the string are the same. More formally, let
   $$\begin{array}{rcl@{\quad}rcl@{\quad}rcl}
   \mathit{left}(\epsilon) & = & 0, & \mathit{middle}(\epsilon) & = & 0, & \mathit{right}(\epsilon) & = & 0, \\
   \mathit{left}(x\mathtt{[}) & = & \mathit{left}(x)+1, & \mathit{middle}(x\mathtt{[}) & = & \mathit{middle}(x), & \mathit{right}(x\mathtt{[}) & = & \mathit{right}(x), \\
   \mathit{left}(x\mathbf{I}) & = & \mathit{left}(x), & \mathit{middle}(x\mathbf{I}) & = & \mathit{middle}(x)+1, & \mathit{right}(x\mathbf{I}) & = & \mathit{right}(x), \\
   \mathit{left}(x\mathtt{]}) & = & \mathit{left}(x), & \mathit{middle}(x\mathtt{]}) & = & \mathit{middle}(x), & \mathit{right}(x\mathtt{]}) & = & \mathit{right}(x)+1
   \end{array}$$
   A string x is in T iff
   $$\forall y, z.\ x = yz.\ (\mathit{left}(y) \geq \mathit{middle}(y)) \wedge (\mathit{middle}(y) \geq \mathit{right}(y)) \wedge (\mathit{left}(x) = \mathit{middle}(x) = \mathit{right}(x))$$
   For example, the strings
   $$\mathtt{[}\mathbf{I}\mathtt{]} \quad \mathtt{[[}\mathbf{I}\mathtt{[}\mathbf{I}\mathtt{]][}\mathbf{I}\mathtt{]}\mathbf{I}\mathtt{]} \quad \mathtt{[}\mathbf{I}\mathtt{[]}\mathbf{I}\mathtt{]}$$
   are in T, but the strings
   $$\mathtt{[][}\mathbf{I}\mathbf{I}\mathtt{]} \quad \mathtt{[]} \quad \mathtt{[}\mathbf{I}\mathtt{[}\mathbf{I}\mathtt{]}\mathbf{I}\mathtt{[]]}$$
   are not.

   (a) **(20 points):** Prove that T is not a context free language.

   (b) **(15 points):** Give the grammars for two CFLs, A_1 and A_2 such that $T = A_1 \cap A_2$.

   (c) **(5 points):** Are context free languages closed under complement? Give a *short* justification for your answer.