1. **(20 points):** Do question 1 from Kozen Homework 4.
   For those who don't have Kozen, I'll copy them into this assignment in a day or two and update the copy on the web.

2. **(20 points):** Do question 2 from Kozen Homework 4.

3. **(15 points):** Do question 3 from Kozen Homework 3.
   Note that this one is from Kozen HW 3, not HW 4 like the first two!

4. **(45 points):** Let $\Sigma = \{0,1\}^3$, i.e. the set of tuples consisting of three bits. For, $(a,b,c) \in \Sigma$, define

$$\begin{aligned} \mathsf{first}((a,b,c)) &= a \\ \mathsf{second}((a,b,c)) &= b \\ \mathsf{third}((a,b,c)) &= c \end{aligned}$$

   We overload first, second, and third to strings as shown below:

$$\begin{aligned} \mathsf{first}(\epsilon) &= \epsilon \\ \mathsf{first}(x \cdot \mathsf{c}) &= \mathsf{first}(x) \cdot \mathsf{first}(\mathsf{c}) \end{aligned}$$

$$\begin{aligned} \mathsf{second}(\epsilon) &= \epsilon \\ \mathsf{second}(x \cdot \mathsf{c}) &= \mathsf{second}(x) \cdot \mathsf{second}(\mathsf{c}) \end{aligned}$$

$$\begin{aligned} \mathsf{third}(\epsilon) &= \epsilon \\ \mathsf{third}(x \cdot \mathsf{c}) &= \mathsf{third}(x) \cdot \mathsf{third}(\mathsf{c}) \end{aligned}$$

   For example, if $s = (0,0,0)(0,0,1)(0,1,0)(0,1,1)(1,0,0)(1,0,1)$, then $\mathsf{first}(s) = 000011$, $\mathsf{second}(s) = 001100$, and $\mathsf{third}(s) = 010101$. For $s \in \{0,1\}^*$, let $\mathsf{binary}(s)$ denote the binary value of s when the most significant bit is the first symbol of the string:

$$\begin{aligned} \mathsf{binary}(\epsilon) &= 0 \\ \mathsf{binary}(\mathsf{0}) &= 0 \\ \mathsf{binary}(\mathsf{1}) &= 1 \\ \mathsf{binary}(x \cdot \mathsf{c}) &= 2 * \mathsf{binary}(x) + \mathsf{binary}(c) \end{aligned}$$

   For each of the languages described below, state whether or not the language is regular. If the language is regular, then construct a DFA, NFA, or pattern for the language to justify your conclusion; you may also use any property of regular languages that we have proven in class or that is proven in the textbook. If the language is not regular, prove that it is not. Again, you may use any property of regular languages that has been proven in class or in the textbook.

   (a) **(15 points):** binary addition

$$B = \{w |\ \mathsf{binary}(\mathsf{third}(w)) = \mathsf{binary}(\mathsf{first}(w)) + \mathsf{binary}(\mathsf{second}(w))\}$$

   (b) **(15 points):** unary numbers

$$\begin{aligned} B = \big\{w |\ \exists m_1, m_2, m_3. \quad & (\mathsf{first}(w) = 1^{m_1}0^{|w|-m_1}) \\ \wedge \quad & (\mathsf{second}(w) = 1^{m_2}0^{|w|-m_2}) \\ \wedge \quad & (\mathsf{third}(w) = 1^{m_3}0^{|w|-m_3})\big\} \end{aligned}$$

   (c) **(15 points):** unary addition

$$\begin{aligned} B = \big\{w |\ \exists m_1, m_2. \quad & (\mathsf{first}(w) = 1^{m_1}0^{|w|-m_1}) \\ \wedge \quad & (\mathsf{second}(w) = 1^{m_2}0^{|w|-m_2}) \\ \wedge \quad & (\mathsf{third}(w) = 1^{m_1+m_2}0^{|w|-(m_1+m_2)})\big\} \end{aligned}$$